8 Porous Media Theory for Membrane Transport Phenomena

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8.1 INTRODUCTION

Hollow fiber dialyzers, that is, artificial kidney systems, are widely used in the therapy of hemodialysis, which is a method for removing waste products such as creatinine and urea, as well as free water from the blood when the kidneys are in renal failure. In this dialyzer, a bundle of hollow fibers of ultrafiltration membranes are used so as to remove metabolic end products from the human body. Mass diffusion and ultrafiltration processes through such membranes are most commonly described by the Kedem–Katchalsky model (1958), which estimates the volume and solute flows of nonelectrolyte solutions across membranes. In this 1D model, the total solute flux vector perpendicular to the membranes is evaluated in consideration of the solute diffusion rate through the membranes due to the difference of the concentration as well as the ultrafiltration flow induced by the transmembrane pressure and osmotic pressure.

Hollow fibers are also used in reverse osmosis modules for water purification. Such hollow fiber reverse osmosis modules are most extensively used for both domestic and industrial water purification, since they are capable of providing high specific surface area and recovery factor. Various membrane transport models are available for mathematically describing transport phenomena through membranes with concentration polarization. Sekino (1993, 1995) was the first to introduce what is known as the friction–concentration–polarization model. The Sekino model was adopted by Al-Bastaki and Abbas (1999, 2000), Kumano et al. (2008), and Marcovecchio et al. (2010) for estimating desalination performance of actual modules. Chatterjee et al. (2004), Gupta (1985, 1987) and Murthy and Gupta (1994, 1998) replaced this concentration polarization by applying the three-parameter, nonlinear, Spiegler–Kedem model (1966) to analyze the performances of radial flow hollow fiber reverse osmosis modules.

In the Spiegler–Kedem model (1966), the so-called reflection coefficient is introduced as another parameter in addition to two parameters commonly used in the solution–diffusion model, namely, hydrodynamic permeability and solute permeability. The Spiegler–Kedem model is believed to be more accurate for describing transport phenomena through membranes with polarization. However, in this Spiegler–Kedem model (1966), the solute flux across the membrane is assumed to be 1D, as in the Kedem–Katchalsky model (1958). Thus, it fails to describe the complex 3D mass and momentum transfer taking place within a reverse osmosis module.

The concept of local volume-averaging theory (VAT) widely used in the study of porous media (Cheng 1978, Vafai and Tien 1981, Quintard and Whitaker 1993, Nakayama 1995, Kuwahara et al. 1996, Nakayama and Kuwahara 2008, Yang and Nakayama 2010) is quite useful under these situations, in which thousands of small-scale elements exist in a large space. In the VAT, we use a grid system, which is just fine enough to resolve macroscopic flow and concentration fields. We do not grid small-scale elements but model them as a porous medium. A variety of models for the fiber membrane systems have been proposed so far. Yet the theory of porous media appears to be most appropriate for treating transport phenomena within hollow fiber systems since it is of 3D and contains fewer assumptions as compared to existing models.

In this chapter, we shall prove that the VAT itself is capable of describing such complex 3D flow and concentration fields prevailing within hollow fiber membrane dialyzer systems and hollow fiber reverse osmosis desalination systems. Therefore, there is no need to prescribe any particular membrane mass transport model. In fact, the VAT in a general 3D form, when applied to a 1D mass transport system, naturally reduces to an appropriate membrane mass transport equation for a hollow fiber devise.

In what follows, a rigorous mathematical development based on the VAT (Sano and Nakayama 2012) is presented to obtain two complete sets of the 3D volume-averaged governing equations appropriate for fiber membrane dialyzer and reverse osmosis desalination systems, respectively. In these models, three individual concentrations are assigned according to the porous media theory for individual phases, namely, the lumen-side fluid phase within the hollow fibers, the shell-side fluid phase outside the hollow fibers, and the membrane phase. This three-concentration equation model derived through the porous media approach can be used to estimate performances of both hollow fiber membrane dialyzers and hollow fiber reverse osmosis modules.

First, we shall present 1D analytic solutions for both hollow fiber dialyzer and reverse osmosis module. We shall compare the analytic results against available experimental data so as to examine the validity of the three-concentration equation model derived through the porous media approach. Subsequently, the 3D numerical computations will be proposed to capture individual concentration fields within a module. The present numerical methods based on the three-concentration model can be used for possible optimization of various hollow fiber systems.

8.2 VOLUME-AVERAGING THEORY

In what follows, the porous media approach, that is, the VAT, is introduced to derive a complete set of the volume-averaged governing equations for the case of fiber membrane dialyzer systems. The corresponding set of the volume-averaged governing equations can easily be obtained for the