
12 Feedback Control for Promoting or Suppressing the Transition to Weak Turbulence in Porous Media Convection

Peter Vadasz

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12.1 INTRODUCTION

Vadasz and Olek (1999a,b, 2000a,b) and Vadasz (1999a,b, 2001a–d, 2002, 2010a) demonstrated theoretically that the transition from steady to chaotic (weak-turbulent) convection in saturated porous media yields a system that is equivalent to Lorenz equations (Lorenz, 1963). Vadasz (1999c, 2006) presented similar results for the corresponding problem of convection in pure fluids and Vadasz and Olek (1998) for centrifugally induced convection in a rotating porous layer (see also Vadasz, 1998; Straughan, 2001).

The linear stability analysis of the Lorenz equations around the nontrivial stationary points provides a critical value of the controlling parameter, the scaled Rayleigh number, $R_o = Ra_o/Ra_{cr}$, representing the transition from steady solutions to chaos, where Ra_{cr} represents the other critical Rayleigh number for the transition from a motionless solution (trivial stationary point) to a steady convection (nontrivial stationary point). A distinction is made here between numerical solutions of the reduced set of equations, such as the Runge–Kutta method, and computational solutions meant to represent basically analytical solutions that are then computed, such as the Adomian decomposition method. Numerical, computational (Vadasz, 1999a–c, 2001a–d, 2002, 2006, 2010b; Vadasz and Olek, 1999a,b, 2000a,b), and experimental results (Wang et al., 1992; Yuen and Bau, 1996) show that the former transition occurs at subcritical values of the Rayleigh number, that is, at $R \leq R_o$. Sparrow (1982) shows that for the Lorenz system analyzed around the origin, the transition to chaos is via a homoclinic explosion and that the homoclinic orbit, which exists just at the point where the solution orbiting around one nontrivial steady solution turns toward the other nontrivial steady solution, belongs to the subcritical Hopf bifurcation obtained at $Ra = Ra_o$.

Mahmud and Hashim (2010) suggested a feedback control system (Bau, 1999) to be applied to the porous media convection based on the porous media feedback control presented by Tang and Bau (1993) and Zhao and Bau (2006). Magyari (2010) demonstrated that the structure of the feedback