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# 21 Thermogravitational Diffusion in a Porous Medium Saturated by a Binary Fluid

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## 21.1 INTRODUCTION

Double-diffusive convection in a saturated porous medium due to temperature and concentration gradients has been widely studied because of its numerous fundamental and industrial applications. Reviews of recent development and publications in this field are given in the books of Nield and Bejan (2012), Ingham and Pop (2000, 2002, 2005), and Vafai (2005).

The linear stability analysis of the flow in a porous medium saturated by a fluid mixture, in the presence of a temperature gradient and taking into account the Soret effect, was investigated by Ouarzazi and Bois (1994). The temperature gradient was assumed to vary periodically with respect to time. The two-dimensional (2D) instability thresholds, for both oscillatory and stationary instabilities, were predicted by these authors. The onset of Soret-driven convection in an infinite horizontal porous layer heated isothermally from below or from above has been considered by Sovran et al. (1996) using a linear stability analysis and nonlinear perturbation theories. A linear stability analysis was used to predict the onset of motion in terms of the buoyancy ratio, the Lewis number, and the normalized porosity. Depending on the value of the separation ratio, it was found analytically and numerically that the motionless solution lost its stability via stationary or Hopf bifurcations. Knobloch (1986), Platten and Legros (1984) studied the onset and development of convection in horizontal rectangular cavity. The two vertical walls of the cavity are impermeable and adiabatic, while the impermeable horizontal walls are maintained at constant and different temperatures. For a given Lewis number,  $Le$ , they obtained the stability diagram in which the critical Rayleigh number is given as a function of the separation ratio  $\psi$ . Bahloul et al. (2003) studied the onset of Soret-driven convection in a shallow porous horizontal layer submitted to a vertical heat flux on the two horizontal walls. The thresholds for finite amplitude, oscillatory, and monotonic convection instabilities are determined in terms of the governing parameters of the problem. For their study, the authors used a closed-form analytical solution based on the parallel flow approximation. However, they were not interested in the separation of the binary mixture components.

This work was extended by Bourich et al. (2004) not only for a shallow horizontal porous cavity but also for a shallow enclosure with a binary fluid. The critical Rayleigh numbers for the onset of oscillatory and stationary convection were determined explicitly as functions of the governing parameters for infinite layers and bounded boxes. At the onset of instability, they found that the wave number was equal to zero, and they also showed the dependency of critical parameters on the normalized porosity. Three-dimensional numerical modeling of Soret-driven convection in a cubic