17 Convection of a Bingham Fluid in a Porous Medium

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CONTENTS

17.1 INTRODUCTION

The chief purpose of this chapter is to consider convective flows of a Bingham fluid when it saturates a porous medium. My interest in this topic arose after seeing the work of Turan et al. (2012) at the *Advances in Computational Heat Transfer Symposium*, which was held at the University of Bath. In that paper, the authors considered the convection of a Bingham fluid in a square cavity heated from a vertical sidewall and cooled by the other. Plots of streamlines show unyielded regions where the local shear stress is less than the yield stress. I immediately wondered what the equivalent would be for a porous medium, and I started searching for papers on the topic. I was assisted in this work by a final year undergraduate who also worked on some network modeling aspects (Nash 2013). Of course, the presence of the solid matrix means that flow in a porous medium will happen only when the yield stress is exceeded locally. If one were, in the first instance, to think of a porous medium as a bundle of tubes or a collection of channels or even a network of channels, then simple considerations of Poiseuille flow or Hagen–Poiseuille flow in these tubes or channels lead one immediately to realize that *unyielded* in the porous medium context actually means *no flow*. This is not necessarily the case for the so-called clear fluid flows, such as Poiseuille flow, where one may have flow within a channel, but the unyielded central portion still moves. Therefore, I have used the opportunity to provide this chapter as one where I would have an extra impetus to study the general topic of Bingham fluids and how they convect in a porous medium.

It has become apparent, at one and the same time, that (1) there is a huge literature associated with yield-stress fluids in general, one that I had no idea existed, and (2) there are very few papers indeed that deal with the convection of a yield-stress fluid in a porous medium even though there is much interest in isothermal flows in porous media. This juxtaposition of glut and famine has therefore guided how I have approached the task of writing. So this chapter begins with a brief, possibly too brief, introduction first to the concept of a yield-stress fluid in general, and second to the modeling of isothermal flows in porous media. There are some controversies that the reader, new to this topic, will, I hope, find useful to know. There are varieties of constituent models and even the questioning of whether there is indeed such a thing as a yield stress. My aim here is not to provide a definitive set of textbook information, but merely to indicate some of the issues that may usefully be pursued.

After a long time spent searching the published literature, I was successful in finding only about a dozen papers on the convection of a yield-stress fluid in a porous medium, and I was surprised to find that all of these are devoted to boundary-layer flows. There was nothing at all on convection in channels, on convection in cavities, or on the analog to the Darcy–Bénard problem. Therefore, much of this chapter concentrates on the new work on these topics and gives a flavor of what to expect in more detailed studies.

After introducing more fully the concept of a yield-stress fluid and of how it is modeled when flowing through a porous medium, the chapter continues with further new studies of isothermal flows in distributions of channels and in square networks. The latter allows the definition of a new type of Darcy–Bingham law for flow in such a structured medium and demonstrates that flow is anisotropic in periodic media. After that, we consider the flow which is induced in a sidewall-heated vertical channel, and it is possible to derive analytical solutions for this case. Nonlinear convection in sidewall-heated cavities are also computed, as are flows of Darcy–Bénard type; in both cases, certain qualitative features that are present for fluids without yield stresses are changed. An unsteady boundary-layer flow is also considered in some detail. Finally, the steady boundary-layer flows mentioned earlier are considered. In this case, we make an argument that there are some difficulties with the classical type of boundary-layer analysis where the subsequent consideration is not made of the flow that occurs in the outer external region. We show that, when the yield criterion lies within the boundary layer itself, then entrainment into the boundary, something that must happen (1) to provide fluid to replace that which is convected upward and (2) to prove a mechanism for restricting the conduction of heat perpendicularly away from the surface, cannot happen because it is too weak to overcome the yield criterion. Thus, this chapter provides a great deal of new information on new topics. It is not comprehensive but provides a foundation for more detailed studies, which I intend to pursue in the next few years.

17.2 YIELD-STRESS FLUIDS AND THEIR MODELING

In this section, we take a brief look at various models of Bingham fluids and how one might then define macroscopic laws for the flow of such fluids in porous media.

17.2.1 Bingham Fluids

A Bingham fluid is essentially a Newtonian fluid but one with a yield stress. Thus, the fluid flows only when a sufficiently large stress is imposed, but thereafter, it flows with a constant