$$V = V_m, \quad \mu \frac{\partial V}{\partial z} = \mu \frac{\partial V_m}{\partial z} \tag{2}$$

$$W = W_m, \quad P + 2\mu \frac{\partial W}{\partial z} = P_m + 2\mu \frac{\partial W_m}{\partial z} \tag{3}$$

in which V and W are horizontal and vertical velocity components in the y and z directions, P is the pressure, μ is the viscosity, and the subscript m denotes quantities pertaining to the porous medium. It is noted that we had chosen the "effective viscosity" in the porous medium to be equal to the fluid viscosity (Neale and Nader, 1974).

By continuity, one can show that $\partial W/\partial z = \partial W_m/\partial z$, and with μ common in both regions, one can conclude from Eq. (3) that $P = P_m$, as shown in Eq. (22) of our paper. Since in the two papers under discussion $\mu_I \neq \mu_{II}$, Eq. (1) as written is not an identity but an extra imposed boundary condition, as pointed out by Nield (1995).

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Authors' Closure

The authors, Chen and Chen (1992), are right in that Eq. (1) is correct when the two viscosities are equal. However, a careful reading would make it very clear that the main subject of the discussion by Nield (1995) was indeed about the fact that Vafai and Huang (1994) and Huang and Vafai (1994) are setting the fluid viscosity and the effective viscosity equal, i.e., they are using the same viscosity in front of both terms in both papers. In fact, Nield (1995) had stated in the very beginning of his discussion about the works of Vafai and Huang (1994) and Huang and Vafai (1994) that, "In each of these papers the authors have modeled flow in a porous medium by a Brinkman–Forchheimer–extended Darcy equation (Eq. (5) of the first paper, Eq. (2) of the second) in which the coefficient of the Darcy term $\mathbf{v}/K$ is the same as the coefficient of the Brinkman term $\nabla^2\mathbf{v}$, and each is denoted by ν_{eff}." Therefore, the primary subject of the discussion and the closure (Vafai and Kim, 1995) is centered around the fact that our coefficient for the Darcy term $\mathbf{v}/K$ is the same as our coefficient for the Brinkman term $\nabla^2\mathbf{v}$! We use different symbols for the fluid viscosity and the effective viscosity so as to make it clear that they are usually different (also to avoid another discussion on the same point), but then we make it clear that due to lack of definitive data we always use the same value of viscosity for both when dealing with porous-fluid interfaces. We cannot emphasize this point any more strongly. It should be noted that our proof is based on facts and is independent of the material referred to by Chen and Chen (1992). We referred to their paper in our closure for the sake of brevity and as another example of the use of the type of equation that we are employing.

Furthermore, Vafai and Kim (1995) had made it extremely clear very early on that they had taken the coefficient for the Darcy term $\mathbf{v}/K$ to be the same as the coefficient for the Brinkman term $\nabla^2\mathbf{v}$ and why we had done so. In fact, in Vafai and Kim (1995) the foremost point starts with the following statement: "In general the coefficient of the Darcy term $\mathbf{v}/K$ is μ_f and the coefficient of the Brinkman term $\nabla^2\mathbf{v}$ is μ_{eff} as shown in Eq. (5) of Vafai and Kim (1990). We are well aware that there are some situations where it is important to make a distinction as shown and discussed, for example, in the works of Vafai and Kim (1990), Ettefagh et al. (1991) and Huang and Vafai (1994). But Lundgren (1972) and Neale and Nader (1974) have shown that setting the effective viscosity of the fluid-saturated porous medium equal to the fluid viscosity provides good agreement with experimental data. Hence, lacking definitive information on μ_{eff} it has become a common practice to set the effective viscosity equal to the fluid viscosity." This effective viscosity as explained by Vafai and Kim and Huang and Vafai several times is taken to be the fluid viscosity. This has always been our de-facto approach for problems dealing with the porous-fluid interface. This is similar to the work of several other investigators as cited by Vafai and Kim (1995) (which are also cited by Chen and Chen, 1992). It should be noted once again that we did not even directly use this condition when we solved our problem numerically. This is because we used a one-domain approach in terms of the vorticity–stream function–temperature formulation, which satisfies this condition indirectly. Finally, we would like to make it clear that Chen and Chen's last argument is wrong on both accounts because in the papers under discussion, as well as in Vafai and Kim (1995), $\mu_I = \mu_{II}$ as we had stated earlier. Hence we did not impose an extra boundary condition as claimed by Nield (1995).

Ironically, it appears that the effective viscosity and the fluid viscosity are not taken to be equal by Chen and Chen (1992). That is, the coefficient for the Darcy term $\mathbf{v}/K$ and the coefficient for the Brinkman term $\nabla^2\mathbf{v}$ are not set to be equal by Chen and Chen (1992). Their non-Darcian governing equation (Eq. (5)), which they have cited based on Georgiadis and Catton (1986), is actually based on the earlier work of Georgiadis and Catton (1985a, 1985b). This non-Darcian governing equation model "... is similar to the one implemented by Vafai and Tien [1] {1981} and Vafai [2] {1984} for forced boundary layer convective flow ..." (Georgiadis and Catton, 1985a, 1985b). Vafai and Tien (1981) obtained the governing momentum equation based on local volume averaging and matched asymptotic expansion. In the work of Vafai and Tien (1981), the well-established empirical information from the core region of the flow was incorporated into the analysis by using the proper matching of the inner and outer flows. This process also replaced the lost information, which is encountered as a result of the local volume averaging method. It should be noted that for this governing equation, as given by Vafai and Tien (1981), the coefficient in front of the Brinkman term $\nabla^2\mathbf{v}$ is $\Lambda_B = \mu_f/\delta$, the coefficient in front of the $\mathbf{v}$ term is $\Lambda_P = \mu_f/K$, the coefficient in front of the ∇P is unity, and the coefficient in front of the convective term (it is shown in Vafai and Tien (1981) that the advective term is negligible and the treatment for the transient part of the convective term, which is not present in Vafai and Tien (1981), is given by Vafai and Tien (1982)) is $\Lambda_I = \rho_f/\delta$. Accounting for the fact that different symbols have been used, these are the same coefficients in front of the corresponding terms of Chen and Chen (1992). Chen and Chen (1992) needed to set the effective viscosity, which refers to the coefficient in front of the Laplacian of the velocity of the fluid saturated porous medium, equal to the fluid viscosity. This point is directly and exactly consistent with the original formulation of Brinkman (1947), as displayed in his Eq. (5), that of Neale and Nader (1974), as displayed in their Eq. (1.7), as well as various other investigators. This is precisely what Vafai and Huang (1994) and Huang and Vafai (1994) have done. However, as it becomes clear in their Eqs. (16) and (17), Chen and Chen (1992) have failed to set these viscosity coefficients equal. This mistake affects the results of the analysis of Chen and Chen (1992).

Again, in both of these papers (Vafai and Huang, 1994; Huang and Vafai, 1994), and as very clearly reflected in the closure (Vafai and Kim, 1995), the coefficient for the Darcy term $\mathbf{v}/K$ is taken to be the same as the coefficient for the Brinkman term $\nabla^2\mathbf{v}$. We would like once again to reiterate that we are not trying to resolve a philosophical and complex question with respect to the physical nature of the interface. In reality, a fluid–fluid or porous–fluid interface is significantly more complicated than what has been modeled by investigators in both fluid–fluid and porous–fluid interface modeling. Here we have adopted the traditional mathematical idealization used for both fluid–fluid and porous–fluid interfaces, i.e., representing the interface by a singular surface. Within this framework the equations used and the results obtained are correct as they stand.

We would like to thank the authors for an extension of this discussion. However, we have already discussed at length and with clarity an inapplicable point regarding a boundary condition at the interface that was never explicitly used to solve this (porous–fluid interface) problem. As such we feel that any additional discussion on this point will not be helpful to the readers of the journal.

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