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A Mathematical Study of Non-Newtonian Micropolar Fluid in Arterial Blood Flow Through Composite Stenosis

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Abstract: The unsteady and incompressible flow of non-Newtonian fluid through composite stenosis is investigated in the present study. The micropolar fluid is treated as a blood flow model. Mild stenosis and slip velocity are also taken into account. The governing equations are given in cylindrical coordinates system. Analytic solutions of velocity and volumetric flow flux are developed interms of modified Bessel functions. The expressions for the impedance (flow resistance) λ, the wall shear stress distribution in the stenotic region $T_{\mathbf{w}}$ and the shearing stress at the stenosis throat $T_{\mathbf{s}}$ are also given. Impact of involved pertinent parameters is sketched and examined by the resistance of impedance and shear stress. The stream lines are also made for different sundry parameters.

Keywords: Micropolar fluid, Blood flow, Stenosed arteries, Exact solutions

1 Introduction

In the last three decades, the study of non-Newtonian fluids [1–7] has gained great importance and this is mainly due to their huge range of applications, such as blood flow, paints, melts of polymers, biological solutions and glues but a careful review of the literature reveals that non-Newtonian micropolar fluid has yet received very little attention. The study of blood flow of non-Newtonian fluids in a stenosed artery is very important because of the fact that number of cardiovascular diseases in the blood vessels such as hearts attacks and strokes are the leading cause of death worldwide. Even though the considerable inventions for the diagnosis and treatment of these disorders have been made, but still this subject needs more care and attention to avoid these diseases. Few investigators have highlighted different aspects of blood flow analysis in arteries.

Recently, Mekheimer and El Kot [8] have studied the mathematical modeling of unsteady flow of such fluids through an anisotropically tapered elastic arteries with time variant overlapping stenosis. They [8] analytically solved their mathematically model for mild stenosis case. Riahi et al [9] have examined the problem of blood flow in an artery in the presence of an overlapping stenosis. A mathematical study on three layered oscillatory blood flow through stenosed arteries have been investigated by Tripathi [10]. In a number has papers, Mekheimer and El kot [11–14] have discussed the different aspects of blood flow analysis in stenosed arteries. Very recently, Mishra et al [15] have studied the blood flow through a composite stenosis in an artery with permeable wall. Some relevant studies on the topic can be seen from the list of references [16, 17] and a number of references on the topic can be found therein.

Moreover, getting an exact analytic solution of a given coupled partial differential equation is often more difficult as compared to getting a numerical solution. However, results obtained by numerical methods give discontinuous points of a curve when plotted. Though numerical and analytic methods for solving coupled partial differential equations have limitations, at the same time they have their own advantages too. Therefore, we cannot neglect either of the two approaches but usually it is pleasing to

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solve a coupled partial differential equations analytically [18–24].

Motivated by these facts, the present work has been undertaken in order to get an exact solution of fully developed flow of an incompressible blood flow of micropolar fluid through a composite stenosis in an artery with permeable wall. Physical problem is first modelled and then simplified by using the nondimensional variables. The analytical solution of the simplified equations are found and the results for the resistance impedance, wall shear stress axial velocity, pressure gradient and stream functions are discussed through graphs for various physical parameters of the problem. After the introduction in Section 1, the outlines of this paper are as follows. Section 2 contains mathematical formulation. In Section 3 exact solutions of the problems are presented. Discussion and results are given in Sections 4. Finally Section 6 summaries the concluding remarks.

2 Mathematical formulation of problem

Consider an incompressible micropolar fluid is flowing through a composite stenosis in a circular artery with permeable walls. We are considering cylindrical coordinates (r,θ,z) in such a way that $z-$axis is taken along the axis of the artery where as r, θ are the radial and circumferential directions respectively (see Fig. 1). Let $r=0$ is considered as the axis of the symmetry of the tube. The geometry of the stenosis is assumed to be manifested in the arterial segment is described as

$$R(z)/R_0 = 1-\frac{2\delta}{R_0\, L_0}(z-d);\; d\le z\le d+L_0/2, \tag{1}$$

$$= 1-\frac{2\delta}{R_0}\left(1+\cos\frac{2\pi}{L_0}(z-d-L_0/2)\right)$$

$$;\quad d+L_0/2\le z\le d+L_0, \tag{2}$$

$$= 1;\;\; otherwise, \tag{3}$$

where $R\approx R(z)$ and R_0 are the radius of the artery with and without stenosis, is respectively, L_0 is the length of the stenosis and d indicates its location, δ is the maximum projection (maximum height) of the stenosis at $z=d+L_0/2$. Consider the blood as micropolar fluid, the equations for the flow model are described as

$$\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial z}=0 \tag{4}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial z}+(\mu+k)\times\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{k}{r}\frac{\partial (r\,G)}{\partial r}, \tag{5}$$

$$\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+(\mu+k)\times\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)-k\frac{\partial\, G}{\partial z}, \tag{6}$$

$$\rho j\left(v\frac{\partial G}{\partial r}+u\frac{\partial G}{\partial z}\right)=-2kG-k\left(\frac{\partial u}{\partial r}-\frac{\partial v}{\partial z}\right)+\gamma\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (rG)}{\partial r}\right)+\frac{\partial^2 G}{\partial z^2}\right). \tag{7}$$

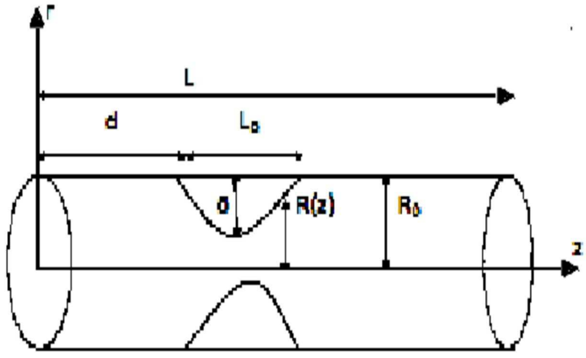

Fig. 1: Flow geometry of a composite stenosis in an artery with permeable wall.

Introducing the following nondimensional variables

$$x'=\frac{x}{b},\; r=\frac{r}{d_0},\; u\prime=\frac{u}{u_0},\; v=\frac{bv}{u_0\delta},\; h'=\frac{h}{d_0},\; p'=\frac{d_0^2 p}{u_0 b\mu},\;\; j'=\frac{j}{d_0^2},\; G'=\frac{d_0 G}{u_0}.$$

Making use of above nondimensional quantities and for the case of mild stenosis, we arrive at

$$\frac{\partial p}{\partial z}=\frac{1}{1-N}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{N}{r}\frac{\partial}{\partial r}(rG)\right), \tag{8}$$

$$2G=-\frac{\partial u}{\partial r}+\frac{2-N}{m^2}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rG)\right), \tag{9}$$

where

$$N=\frac{k}{\mu+k},\; m^2=\frac{d_0^2 k(2\mu+k)}{\gamma(\mu+k)}. \tag{10}$$

The boundary conditions are

$$\frac{\partial u}{\partial r}=0 \text{ at } r=0, \tag{11}$$

$$u=u_B \text{ and } \frac{\partial u}{\partial r}=\frac{\alpha}{\sqrt{D_\alpha}}(u_B-u_{porous}) \text{ at } r=R(z), \tag{12}$$

where $u_{porous}=-\frac{D_a}{\mu}\frac{\partial p}{\partial z}$, u_{porous} is the velocity in the permeable boundary, D_a is the Darcy number and α (called the slip parameter) is a dimensionless quantity depending on the material parameters which characterize the structure of the permeable material with in the boundary region.

3 Solution of the problem

Equation (8) can be written as

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}+NrG-(1-N)\frac{r^2}{2}\frac{\partial p}{\partial z}\right)=0. \tag{13}$$

After integrating, we get

$$\frac{\partial u}{\partial r}=(1-N)\left(\frac{r}{2}\frac{\partial p}{\partial z}+\frac{A}{r}\right)-NG, \tag{14}$$

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where A is constant of integration. Invoking Eq. (14) into Eq. (9), we arrive at

$$\frac{\partial^2 G}{\partial r^2}+\frac{1}{r}\frac{\partial G}{\partial r}-(m^2+\frac{1}{r^2})G=\frac{(1-N)m^2}{(2-N)}\left(\frac{r}{2}\frac{\partial p}{\partial z}+\frac{A}{r}\right). \tag{15}$$

The general solution of above equation is defined as

$$G=BI_1(mr)+CK_1(mr)-\frac{1-N}{2-N}\left(\frac{r}{2}\frac{\partial p}{\partial r}+\frac{A}{r}\right), \tag{16}$$

where $BI_1(mr)$ and $CK_1(mr)$ are modified Bessel functions of first order of first and second kind respectively. Substituting Eq. (16) into Eq. (14) and integrating with the help of boundary conditions, we obtain

$$u=\left(\frac{1-N}{2-N}\right)\left(\frac{-\partial p}{\partial z}\right)\left(R^2-r^2-\frac{NR}{m}\left(\frac{I_0(mR)-I_0(mr)}{I_1(mR)}\right)\right)+u_B. \tag{17}$$

The slip velocity u_B is calculated as

$$u_B=\frac{-\sqrt{D_a}(1-N)}{2\alpha}\left(\frac{\partial p}{\partial z}\right)\left(R-\frac{2\alpha\sqrt{D_a}}{1-N}\right). \tag{18}$$

The volumetric flow flux, Q is thus calculated as

$$Q=2\pi\int_0^R rudr,$$

or

$$Q=\frac{\pi(1-N)}{4(2-N)}\frac{\partial p}{\partial z}R^2\left(\begin{array}{c}\frac{2\sqrt{D_a}(N-1)(N-2)R+4D_a(N-1)+(N-1)R^2\alpha}{(N-1)\alpha}\\ +\frac{4N}{m^2}-\frac{2NI_0(mR)}{mI_1(mR)}R\end{array}\right). \tag{19}$$

Note that when $N\rightarrow 0$, the results of Mishra et al [15] can be recovered as a special case of our problem

$$Q=\frac{\pi(1-N)}{4(2-N)}\frac{\partial p}{\partial z}F(z), \tag{20}$$

$$F(z)=R^2\left(\begin{array}{c}\frac{2\sqrt{D_a}(N-1)(N-2)R+4D_a(N-1)+(N-1)R^2\alpha}{(N-1)\alpha}\\ +\frac{4N}{m^2}-\frac{2NI_0(mR)}{mI_1(mR)}R\end{array}\right), \tag{21}$$

$$\nabla p=\int_0^L\left(-\frac{\partial p}{\partial z}\right), \tag{22}$$

$$\lambda=\frac{4(2-N)}{\pi(1-N)}\Psi, \tag{23}$$

where

$$\Psi=\int_0^d\frac{1}{F(z)_{R/R_0=1}}dz+\int_d^{d+L_0/2}\frac{1}{F(z)_{R/R_0\ from(1)}}dz+\int_{d+L_0/2}^{d+L_0}\frac{1}{F(z)_{R/R_0\ from(2)}}dz+\int_{d+L_0}^{L}\frac{1}{F(z)_{R/R_0=1}}dz. \tag{24}$$

The expressions for the impedance (flow resistance) λ, the wall shear stress distribution in the Stenotic region $T_{\mathbf{w}}$ and the shearing stress at the stenosis throat $T_{\mathbf{s}}$ in their non-dimensional form can be calculated as

$$\lambda=1-\frac{L_0}{L}+\frac{\eta}{L}\int_d^{d+L_0/2}\frac{dz}{a^4\left(1+\frac{2\sqrt{D_a}(N-2)}{\alpha R_0^2a^2}\left(R_0a+\frac{2\alpha\sqrt{D_a}}{N-1}\right)+\frac{4N}{a^2m^2R_0^2}\right)}$$

$$+\frac{\eta L_0}{2\pi L}\int_0^{\pi}\frac{1}{\theta^4\left(1+\frac{2\sqrt{D_a}(N-2)}{\alpha R_0^2\theta^2}\left(R_0\theta+\frac{2\alpha\sqrt{D_a}}{N-1}\right)+\frac{4N}{\theta^2m^2R_0^2}\right)}, \tag{25}$$

$$T_{\mathbf{w}}=\frac{1}{\left((R_0/R)^3+\frac{2\sqrt{D_a}(N-2)}{\alpha R_0^2}\left(R_0(R/R_0)^2+\frac{2\alpha\sqrt{D_a}(R/R_0)}{N-1}\right)\right)}, \tag{26}$$

$$T_{\mathbf{s}}=\frac{1}{\left(b^3+\frac{2\sqrt{D_a}(N-2)}{\alpha R_0^2}\left(R_0b^2+\frac{2\alpha\sqrt{D_a}b}{N-1}\right)\right)}, \tag{27}$$

where

$$a\approx a(z)=1-2\delta(z-d)/R_0L_0,\ b=1-\delta/R_0,\ c=-\delta/2R_0,$$

$$\theta\approx\theta(\beta)=b+\cos\beta,\ \beta=\pi-(2\pi/L)(z-d-L_0/2),\ \eta=1+\frac{2\sqrt{D_a}(N-2)}{\alpha R_0^2}\left(R_0+\frac{2\alpha\sqrt{D_a}}{N-1}\right).$$

4 Discussion and results

To observe the quantitative effects of the micropolar parameter m. The coupling number N, computer codes are developed for the numerical evaluation of the analytical results obtain for flow impedance λ, the wall shear stress $T_{\mathbf{w}}$, for parameters values $L=1,2,3$, $L_0=1$, $d=0$ and for various values of slip parameter α, darcy number $\sqrt{D_a}$ and stenosis height δ/R_0. It is noticed from Fig. 2 that the flow impedance λ increases by increasing the stenosis height δ/R_0, while a decrease in impedance is observed by increasing the slip parameter α and tube length L by keeping all other parameter fixed. The effects of m, stenosis height δ/R_0 and Darcy's number $\sqrt{D_a}$ on the impedance λ are plotted in Fig. 3. It is depicted that impedance decreases with the increase in m. Moreover, impedance increases with the increases in stenosis height δ/R_0 and Darcy's number $\sqrt{D_a}$. The effects of tube length L and micro-rotation viscosity N on the impedance are shown in Fig. 4. It is observed that impedance decreases with the increase of both L and N. The effects of impedance against m for various values of N are sketched in Fig. 5. It is seen that with the increase in N impedance decreases for all values of m The wall shear stress $T_{\mathbf{w}}$ against axial distance z for various values of α and $\sqrt{D_a}$ are given in Fig. 6. It is concluded that the wall shear stress increases with the increase in α and decreases with the increase in $\sqrt{D_a}$. The wall shear stress $T_{\mathbf{w}}$ for different values of m and N are shown in Fig. 7. It is seen

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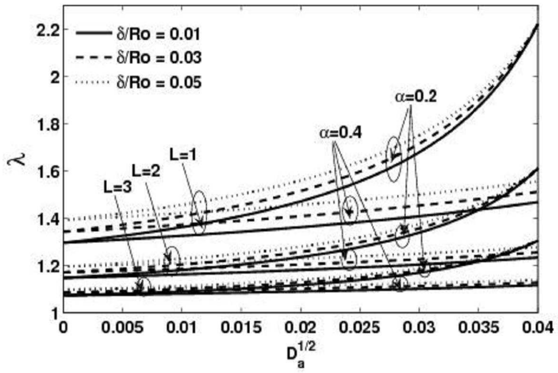

Fig. 2: variation of resistance to flow λ with $\sqrt{D_a}$.

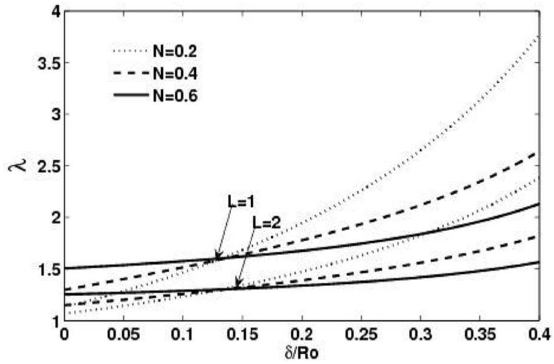

Fig. 4: variation of resistance to flow λ withδ/R_0.

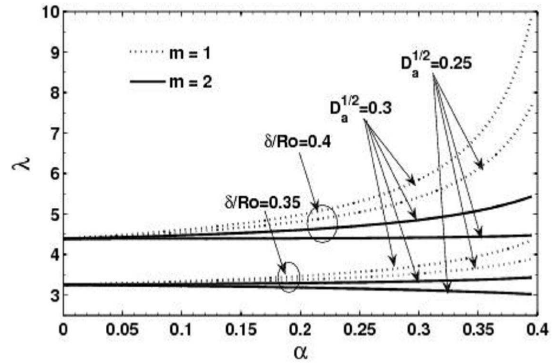

Fig. 3: variation of resistance to flow λ withα.

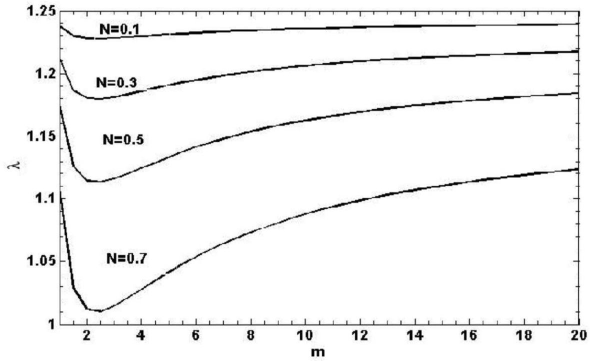

Fig. 5: variation of resistance to flow λ with m.

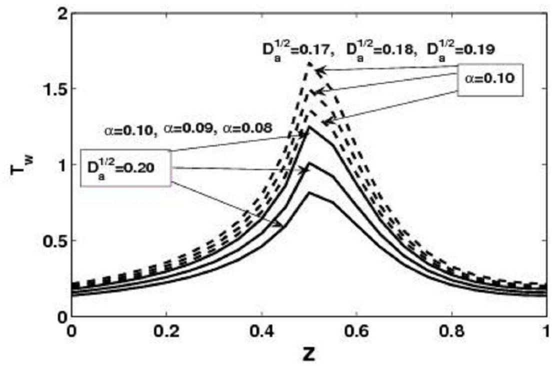

Fig. 6: variation of T_w with z for various values of α and $\sqrt{D_a}$.

that shear stress $T_{\mathbf{w}}$ increases by increasing N and decreasing m. The streams lines for different parameters are shown in Figs. 8 to 11. It is seen that with the increases in m, the size of trapping bolus decreases (see Fig. 8). Fig. 9 shows the streams lines for different values of N. It is observed that with the increases in N, the size of trapping bolus slightly decreases. The streams lines for different values of α are given in Fig. 10. Here the trapping bolus increases with the increases in α. The large Darcy's number $\sqrt{D_a}$, causes to decrease the trapping bolus (see Fig. 11).

5 Conclusion

A mathematical model for the blood flow of micropolar fluid through a composite stenosis in an artery with permeable wall has been studied. The physical problem is first modelled and then simplified by using the nondimensional variables. The exact solution of the governing equations are solved analytically. The results for the resistance impedance, wall shear stress, axial velocity, pressure gradient and stream functions are discussed through graphs for various physical parameters of the problem. To the best of our knowledge, no such analysis is available in the literature which can describe the effects of non-Newtonian micropolar fluid in arterial blood flow through composite stenosis simultaneously.

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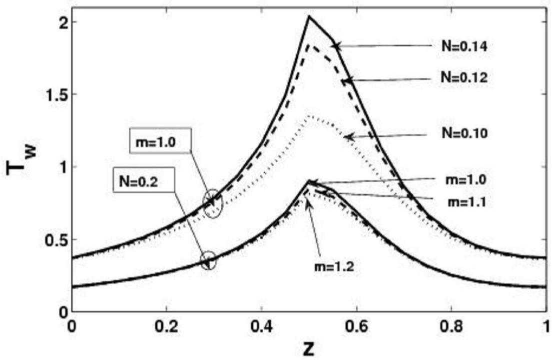
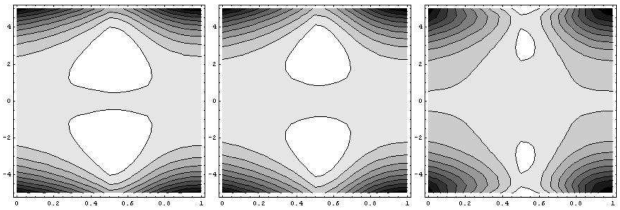
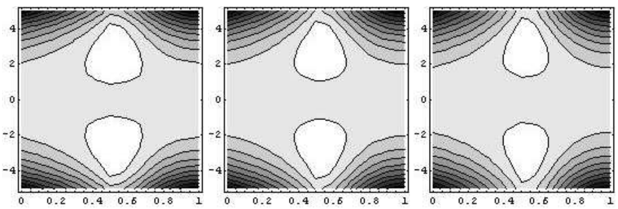


Fig. 7: variation of T_w with z for various values of m and N.



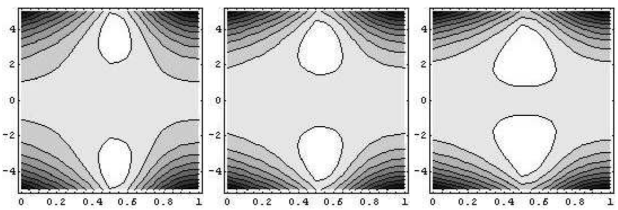
a b c

Fig. 8: Stream lines for (a) $m = 0.5$, (b) $m = 1.0$, (c) $m = 1.4$.



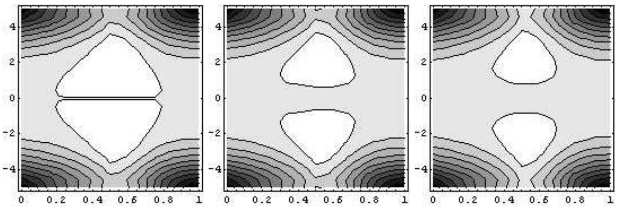
a b c

Fig. 9: Stream lines for (a) $N = 0.1$, (b) $N = 0.2$, (c) $N = 0.3$.



a b c

Fig. 10: Stream lines for (a) $\alpha = 0.03$, (b) $\alpha = 0.04$, (c) $\alpha = 0.05$.



a b c

Fig. 11: Stream lines for (a) $\sqrt{D_a} = 0.34$, (b) $\sqrt{D_a} = 0.36$, (c) $\sqrt{D_a} = 0.38$.

The results presented in this paper are now available for further experimental verification to give confidence for the well-posedness of this boundary value problem. The following main results are observed.

–The stenosis itself and the tapering effect change the flow pattern.
–The flow impedance λ increases by increasing the stenosis height δ/R_0, while a decrease in impedance is observed by increasing the slip parameter α and tube length L.
–Impedance decreases with the increase in m.
–With the increase in N impedance decreases for all values of m.
–The shear stress $T_{\mathbf{w}}$ increases by increasing N and decreasing m.
–By increasing m, the size of trapping bolus decreases.
–Trapping bolus increases with the increases in α.
–The large Darcy's number $\sqrt{D_a}$, causes to decrease the trapping bolus.
–When $N \to 0$, the results of Mishra et al [15] are recovered as a special case of our problem.

References

[1] R. Ellahi, A. Riaz and S. Sohail, Series Solutions of Magnetohydrodynamic Peristaltic Flow of a Jeffrey Fluid in Eccentric Cylinders, J. of Appl. Math. & Inf. Sci., **7**, 1439 (2013).

[2] A. Zeeshan and R. Ellahi, Series solutions of nonlinear partial differential equations with slip boundary conditions for non-Newtonian MHD fluid in porous space, J. of Appl. Math. & Inf. Sci., **7**, 253 (2013).

[3] F. M. Mahomed and T. Hayat, Note on an exact solution for the pipe flow of non-Newtonian third grade fluid. Acta Mech, **190**, 233 (2007).

[4] M. Hameed and S. Nadeem, Unsteady MHD flow of a non-Newtonian fluid on a porous plate, J. of Math. Anal. and Appl., **325**, 724 (2007).

[5] W. C. Tan and T. Masuoka, Stokes' first problem for a second grade fluid in a porous half space with heated boundary, Int. J. Non-Linear Mech., **40**, 512 (2005).

[6] W. C. Tan and T. Masuoka, Stability analysis of a Maxwell fluid in a porous medium heated from below, Phys. Lett. A., **360**, 454 (2007).

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[7] W. C. Tan and T. Masuoka, Stokes first problem for an Oldroyd−B fluid in a porous half space. Phys. Fluids, **17**, 023101 (2005).
[8] Kh. S. Mekheimer and M. A. Elkot, Mathematical modelling of unsteady flow of a Sisko fluid through an anisotropically tapered elastic arteries with time-variant overlapping stenosis, Appl. Math. Model. **36**, 5393 (2012).
[9] D. N. Riahi, R. Roy and S. Cavazos, On arterial blood flow in the overlapping stenosis, Math. & Comp. Model., **54**, 2999 (2011).
[10] D. Tripathi, A mathematical study on three-layered oscillatory blood flow through stenosed arteries,. J. Bionic Eng., **9**, 119 (2012).
[11] Kh. S. Mekheimer and M. A. Elkot, The micropolar fluid model for blood flow through stenotic arteries, Int. J. Pur. Appl. Math., **4**, 393 (2007).
[12] Kh. S. Mekheimer and M. A. Elkot, Suspension model for blood flow through arterial catheterization, Chem. Eng. Comm., **197**, 1 (2010).
[13] Kh. S. Mekheimer and M. A. Elkot, The micropolar fluid model for blood flow through a tapered arteries with a stenosis. Acta Mech. Sinica, **24**, 637–644 (2008).
[14] Kh. S. Mekheimer and M. A. Elkot, Influence of Magnetic field and Hall currents on blood flow through stenotic artery,.Appl. Math. Mech. Eng. Ed., **29**, 1093 (2008).
[15] S. Mishra, S. U. Siddiqui and A. Medhavi, Blood flow through a composite stenosis in an artery with permeable wall, Application and Appl. Math., **6**, 1798 (2011).
[16] N. S. Akber and S. Nadeem, Simulation of heat and chemical reactions on Reiner Rivlin fluid model for blood flow through a tapered artery with a stenosis, Heat and Mass Transf., **46**, 531 (2010).
[17] S. Nadeem, S. Abbasbandy and M. Hussain, Series solutions of boundary layer flow of a micropolar fluid near the stagnation point towards a shrinking sheet , Zeitschrift fur Naturforschung, **64**a, 575 (2009).
[18] S. Chung and K. Vafai, Low-density lipoprotein transport within a multi-layered arterial wall—Effect of the atherosclerotic plaque/stenosis, J. of Biomechanics, **46**, 574 (2013).
[19] L. Ai and K. Vafai, A coupling model for macromolecule transport in a stenosed arterial wall, Int. J. of Heat and Mass Transf., **49**, 1568 (2006).
[20] R. Ellahi, Exact solutions of flows of an Oldroyd 8-constant fluid with nonlinear slip conditions, Zeitschrift Fur Naturforschung A , **65**(a), 1081 (2010).
[21] R. Ellahi, The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe: analytical solutions, Appl. Math. Model., **37**, 1451 (2013).
[22] S. Chung and K. Vafai, Mechanobiology of Low-Density Lipoprotein Transport within an Arterial Wall-Impact of Hyperthermia and Coupling Effects, J. of Bio Mech.
[23] S. Chung and K. Vafai, Effect of the Fluid-Structure Interactions on Low-Density Lipoprotein within a Multi-Layered Arterial Wall, J. of Bio Mech., **45**, 371 (2012).
[24] N. Yang and K. Vafai, Modeling of Low-Density Lipoprotein (LDL) Transport in the Artery- Effects of Hypertension, Int. J. f Heat and Mass Transfer, **49**, 850 (2006).

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