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Effects of heat and mass transfer on peristaltic flow in a non-uniform rectangular duct



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R. Ellahi^{a,b,*}, M. Mubashir Bhatti^b, K. Vafai^a

^a Department of Mechanical Engineering, Bourns Hall, University of California Riverside, CA 92521, USA ^b Department of Mathematics and Statistics, FBAS, IIU, 44000 Islamabad, Pakistan

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ABSTRACT

The model of bioheat transfer in tissues has attracted many researchers due to its application in thermotherapy and human thermoregulation system. Currently bioheat is considered as a heat transfer in human body. In view of this the influence of heat and mass transfer on peristaltic flow in a non-uniform rectangular duct is studied under the consideration of long wavelength ($0 \ll \lambda \rightarrow \infty$) and low Reynolds number ($Re \rightarrow 0$). The flow is examined in wave frame of reference moving with the velocity *c*. Mathematical modeling is based upon the laws of mass, linear momentum, energy and concentration. Analysis is also presented for Prandtl number Pr, Eckert number *E*, Schmidt number *Sc* and Soret number *Sr*. The influence of various emerging parameters of interest is seen for both two and three dimensional graphs. Numerical integration is used to analyze the novel features of volumetric flow rate, average volume flow rate, instantaneous flux and pressure gradient. The trapping bolus phenomena is also presented through stream lines.

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1. Introduction

It is now well established fact that most of the physiological fluids are non-Newtonian in character [1–10]. Several models have been proposed to explain such physiological fluids in order to find the treatment of diagnostic problems that arise during the circulation in a human body. Practical applications and academic curiosity in physiology have generated a lot of interest in studying the peristaltic motion in ducts. Peristaltic mechanism deals with the fluid transport that occurs by a progressive wave of area of contraction or expansion along a length of tubes/channels. Peristaltic transport is found in living body such as movement of ovum in female fallopian tube, swallowing food through esophagus, transport of lymph in lymphatic vessels vasomation of small blood vessels like arterioles, venules, capillaries, transport of spermatozoa in ducts efferentes of male reproductive tract etc. After the experimental work of Latham [11] on peristaltic transport, several theoretical studies [12-20] have been undertaken by many researchers on peristaltic motion under one or more simplified assumptions of small amplitude ratio, small wave number, low Reynolds number and long wavelength etc.

Peristaltic flow with heat and mass transfer has many applications in biomedical sciences and industry such as conduction in

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tissues, heat convection due to blood flow from the pores of tissues and radiation between environment and its surface, food processing and vasodilation. The processes of oxygenation and hemodialysis have also been visualized by considering peristaltic flows with heat transfer. Obviously there is a certain role of mass transfer in all these processes. Mass transfer is important phenomenon in diffusion process such as nutrients diffuse out from the blood to neighboring tissues. Mass transfer also occurs in many industrial processes like membrane separation process, reverse osmosis, distillation process, combustion process and diffusion of chemical impurities. When the effects of heat and mass transfer are considered simultaneously then the complicated relationships occur between driving potentials and fluxes. The energy flux is induced by temperature gradient. The composition gradients and mass flux can be produced by temperature gradient which is known as Soret effect.

Investigations of heat and mass transfer in peristalsis yet have been considered by few researchers. For instance, The influence of heat transfer and magnetic field on peristaltic transport of Newtonian fluid in a vertical annulus has been discussed by Mekheimer and elmaboud [21]. Ellahi et al. [22] investigated the series solutions for magnetohydrodynamic flow of non-Newtonian nanofluid and heat transfer in coaxial porous cylinder with slip conditions. Influence of heat and mass transfer on peristaltic flow of Eyring-Powell and Jeffrey's fluids are discussed by Akbar and Nadeem [23,24]. Radhakrishnamacharya and Murty [25] studied the heat transfer on the peristaltic transport in a non-uniform channel.

^{*} Corresponding author at: Department of Mechanical Engineering, Bourns Hall, University of California Riverside, CA 92521, USA. Tel.: +1 951 275 6747. *E-mail addresses*: rellahi@engr.ucr.edu, rahmatellahi@yahoo.com (R. Ellahi).

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Vajravelu et al. [26] reported the peristaltic flow and heat transfer in a vertical annulus under long wavelength approximation. A careful review of the literature reveals that a very little efforts are yet devoted to examine the peristaltic flow in a rectangular duct. Some relevant studies on the topic can be found from the list of references [27–29].

To the best of our knowledge, no attempt is made to investigate the peristaltic flow in a non-uniform rectangular duct with heat and mass transfer so far. Such consideration is very important since the heat transfer in human tissues involves complicated processes like heat transfer due to perfusion of arterialvenous blood through the pores of tissue, heat conduction in tissues, external interactions and metabolic heat generation such as electromagnetic radiation emitted from cell phones etc. Moreover, it is well known that heat and mass transfer problem in the presence of chemical reaction is very significant in the processes of geothermal reservoirs, drving, enhanced oil recovery, flow in a desert cooler, cooling of nuclear reactors thermal insulation and evaporation at the surface of a water body. These types of flows involve many practical operations such as molecular diffusion of species in presence of chemical reaction within or at boundary. Heat and mass transfer effects are also encountered in chemical industry like in the study of hot salty springs in sea, in thermal recovery processes and in reservoirs. The results obtained for title problem reveal many interesting behaviors that warrant further study on heat and mass transfer problems with chemical reaction.

The flow modeling is based on continuity, momentum and energy equations. These equations are first expressed in terms of stream function and then solved in closed form when the long wavelength and small Reynolds number assumption hold. The effects of magnetic field are taken into account. The obtained expressions are utilized to discuss the role of emerging parameters on the flow quantities. Numerical computations has been used to evaluate the expression for pressure rise. Stream lines of various interesting parameters. Finally, the effect of various emerging parameters are discussed through graphs and trapping phenomenon. This paper is arranged as follows. Section two presents the mathematical formulation for the problem of interest. Section three deals with the solution of the problem. Finally section four synthesis detailed computational results and discussion with the physical interpretation of our findings.

2. Mathematical formulation

We consider the peristaltic flow of an incompressible viscus fluid in a non-uniform duct of rectangular cross section having channel width 2d and height 2a + 2kX. We choose Cartesian coordinate system in such a way that *X*-axis is taken along the axial direction, *Y*-axis is taken along the lateral direction and *Z*-axis is along the vertical direction of channel. The flow geometry of the problem [30] presented as Fig. 1.

Sinusoidal waves of long wavelength are assumed to travel with the velocity c along the walls of channel. These waves are



Fig. 2. Velocity profile for different values of *Q* for fixed $K^* = 0.9$, $\beta = 1.2$, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 3. Velocity profile for different values of β for fixed $K^* = 0.9$, Q = 2, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 4. Temperature distribution for different values of Q for fixed $K^* = 0.9$, $\beta = 0.45$, E = 1, Pr = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 5. Temperature distribution for different values of Pr at $K^* = 0.9$, $\beta = 0.45$, E = 1, $\phi = 0.6$, Q = 1 (a) for 2-dimensional (b) for 3-dimensional.

physiologically from neuromuscular properties of any tubular smooth muscle. From the physical point of view, this is taken care of by an elastic foundation in the mathematical model that before peristaltic motion starts, these walls have to support the hydrostatic pressure *P*. The configuration of the wall surface is given by

$$Z = H(X, t) = \pm a \pm kX \pm b \sin\left[\frac{2\pi}{\lambda}(X - ct)\right],$$
(1)

where *a* and *b* are amplitudes of the waves, λ is wavelength, *t* is time and *X* represents the direction of wave propagation. The walls



Fig. 6. Temperature distribution for different values of *E* at $K^* = 0.9$, $\beta = 0.45$, Pr = 1, $\phi = 0.6$, Q = 1 (a) for 2-dimensional (b) for 3-dimensional.



Fig. 7. Temperature distribution for different values of β at $K^* = 0.9$, $\beta = 0.45$, E = 1, Pr = 1, $\phi = 0.6$, Q = 1 (a) for 2-dimensional (b) for 3-dimensional.



Fig. 8. Concentration distribution for different values of β at $K^* = 0.9$, Pr = 2, E = 1, Sc = 1, Sr = 1, Q = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.

parallel to XZ-plane are not interrupted so here peristaltic wave motion do not contribute. Moreover, the lateral velocity is zero as there is no change in lateral direction of the duct cross section.

In laboratory frame (X, Y) the governing equations, which describe the flow of an incompressible viscous fluid with heat and mass transfer are given as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} = 0, \tag{2}$$

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + W\frac{\partial U}{\partial Z}\right) = -\frac{\partial P}{\partial Z} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2},\tag{3}$$

$$\mathbf{0} = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2},\tag{4}$$

$$\rho\left(\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial X} + W\frac{\partial W}{\partial Z}\right) = -\frac{\partial P}{\partial Z} + \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2},\tag{5}$$



Fig. 9. Concentration distribution for different values of Sr at $K^* = 0.9$, Pr = 2, E = 1, $\beta = 0.45$, Sc = 1, Q = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 10. Concentration distribution for different values of Sc at $K^* = 0.9$, Pr = 2, E = 1, $\beta = 0.45$, Sr = 1, Q = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 11. Concentration distribution for different values of *Q* at $K^* = 0.9$, Pr = 2, E = 1, $\beta = 0.45$, Sc = 1, Sr = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.

$$\begin{aligned} \zeta \left(\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + W \frac{\partial T}{\partial Z} \right) &= \frac{\kappa}{\rho} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} + \frac{\partial^2 T}{\partial Z^2} \right) \\ &+ \nu \left\{ 2 \left[\left(\frac{\partial U}{\partial X} \right)^2 + \left(\frac{\partial W}{\partial Z} \right)^2 \right] + \left(\frac{\partial U}{\partial Y} \right)^2 + \left(\frac{\partial W}{\partial Y} \right)^2 + \left(\frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right)^2 \right\}, \end{aligned}$$
(6)

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + W \frac{\partial C}{\partial Z} = D \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} + \frac{\partial^2 C}{\partial Z^2} \right) + \frac{D K_T}{T_m} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} + \frac{\partial^2 T}{\partial Z^2} \right),$$
(7)

in which ρ is the fluid density, ζ is the specific heat at constant volume, v is the kinematic viscosity of the fluid, κ is the thermal



Fig. 12. Concentration distribution for different values of Pr at $K^* = 0.9$, E = 1, $\beta = 0.45$, Sc = 1, Sr = 1, Q = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 13. Concentration distribution for different values of *E* at $K^* = 0.9$, Pr = 2, $\beta = 0.45$, Sc = 1, Sr = 1, Q = 1, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.



Fig. 14. Variation of dp/dx with x for different values of β at $\phi = 0.7$, $K^* = 0.45$, Q = 0.05.

conductivity of the fluid, *D* is the coefficient of mass diffusivity, T_m is the mean temperature, K_T is the thermal diffusion ratio, *T* and *C* are temperature and concentration of the fluid, respectively.

Under the assumptions that the channel length is an integral multiple of the wave length λ and the pressure difference across the ends of the channel is a constant, the flow is inherently unsteady in the laboratory frame (X, Y) and become steady in the wave frame (x, y) which is moving with velocity 'c' along the wave. The transformation between these two frames is given by



Fig. 15. Variation of dp/dx with x for different values of K^* at $\phi = 0.65$, $\beta = 1$, Q = 0.01.

$$x = X - ct$$
, $y = Y$, $z = Z$, $u = U - c$, $w = W$, $p(x, z) = P(X, Z, t)$.
(8)

where U, V, P are the velocity components, pressure in the laboratory frame and u, v, p are the velocity components, pressure in the wave frame, respectively.

The variables are rendered dimensionless by using the following quantities



Fig. 16. Variation of dp/dx with x for different values of ϕ at $K^* = 0.65, \ \beta = 1, \ Q = 0.01.$



Fig. 17. Variation of dp/dx with x for different values of Q at $K^* = 0.5$, $\beta = 0.9$, $\phi = 0.7$.



Fig. 18. Variation of Δp with Q for different values of β at $K^* = 0.5$, $\phi = 0.7$.

$$\overline{x} = \frac{x}{\lambda}, \quad \overline{y} = \frac{y}{d}, \quad \overline{z} = \frac{z}{a}, \quad \overline{u} = \frac{u}{c}, \quad \overline{w} = \frac{w}{c\delta}, \quad \overline{t} = \frac{ct}{\lambda},$$

$$\overline{h} = \frac{H}{a}, \quad \overline{p} = \frac{a^2 p}{\mu c \lambda}, \quad Re = \frac{\rho a c}{\mu}, \quad \delta = \frac{a}{\lambda}, \quad \phi = \frac{b}{a}, \quad K^* = \frac{k}{a},$$

$$\beta = \frac{a}{d}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \varphi = \frac{C - C_0}{C_1 - C_0}$$

$$(9)$$

Using the above non-dimensional quantities in Eqs. (2)-(7), the equations governing the flow after dropping the bars become

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \mathbf{0},\tag{10}$$

$$Re \ \delta\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \delta\frac{\partial^2 u}{\partial x^2} + \beta^2\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2},\tag{11}$$



Fig. 19. Variation of Δp with Q for different values of ϕ at $\beta = 0.5$, $K^* = 0.5$.



Fig. 20. Variation of Δp with *Q* for different values of K^* at $\phi = 0.5$, $\beta = 0.5$.

$$\mathbf{0} = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial^2 \nu}{\partial x^2} + \delta^2 \frac{\partial^2 \nu}{\partial y^2} + \delta \frac{\partial^2 \nu}{\partial z^2},\tag{12}$$

$$Re \ \delta^2 \left(u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \delta^2 \frac{\partial^2 w}{\partial x^2} + \delta^2 \frac{\partial^2 w}{\partial y^2} + \delta^2 \frac{\partial^2 w}{\partial z^2}, \quad (13)$$

$$\begin{aligned} \operatorname{Re}\,\delta\left(u\frac{\partial\theta}{\partial x}+\delta w\frac{\partial\theta}{\partial z}\right) &= -\frac{k}{\zeta v\rho} \left(\delta^{2}\frac{\partial^{2}\theta}{\partial x^{2}}+\beta^{2}\frac{\partial^{2}\theta}{\partial y^{2}}+\frac{\partial^{2}\theta}{\partial z^{2}}\right) \\ &+\frac{c^{2}}{\zeta(T_{1}-T_{0})} \left[\delta^{2}\left(\frac{\partial w}{\partial z}\right)^{2}+\beta^{2}\left(\frac{\partial u}{\partial y}\right)^{2}+2\delta^{2}\left(\frac{\partial u}{\partial x}\right)^{2}+2\delta^{2}\left(\frac{\partial w}{\partial y}\right)^{2}+\left(\frac{\partial u}{\partial z}+\delta^{2}\frac{\partial w}{\partial x}\right)^{2}\right], \end{aligned} \tag{14}$$

$$Re \ \delta\left(u\frac{\partial\varphi}{\partial x} + w\delta\frac{\partial\varphi}{\partial z}\right) = -\frac{D\rho}{\mu}\left(\delta^{2}\frac{\partial^{2}\varphi}{\partial x^{2}} + \beta^{2}\frac{\partial^{2}\varphi}{\partial y^{2}} + \frac{\partial^{2}\varphi}{\partial z^{2}}\right) + \frac{\rho DK_{T}(T_{1} - T_{0})}{\mu T_{m}(C_{1} - C_{0})}\left(\delta^{2}\frac{\partial^{2}\theta}{\partial x^{2}} + \beta^{2}\frac{\partial^{2}\theta}{\partial y^{2}} + \frac{\partial^{2}\theta}{\partial z^{2}}\right),$$
(15)

in which

$$Pr = \frac{\zeta v \rho}{k}, \quad E = \frac{c^2}{\zeta (T_1 - T_0)}, \quad Sr = \frac{\rho D K_T (T_1 - T_0)}{\mu T_m (C_1 - C_0)}, \quad Sc = \frac{\mu}{D\rho},$$
(16)

where Re is the Reynolds number, δ is the dimensionless wave number, Pr is the Prandtl number, *E* is the Eckert number, *Sc* and *Sr* are the Schmidt and Soret numbers, respectively.

Using long wave length and low Reynolds number approximation Eqs. (11)-(15)reduce to

$$\frac{dp}{dx} = \beta^2 \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2},\tag{17}$$



Fig. 21. Stream lines for different values of β , (a) for $\beta = 0.1$, (b) for $\beta = 0.4$, (c) for $\beta = 0.6$, (d) for $\beta = 0.8$. The other parameters are $K^* = 0.4$, $\phi = 0.45$, Q = 0.01, y = 0.9.

$$-\Pr E\left[\beta^2 \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] = \beta^2 \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2},\tag{18}$$

$$\mathbf{0} = \frac{1}{Sc} \left(\beta^2 \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) + Sr \left(\beta^2 \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right). \tag{19}$$

The expression of stream function satisfying Eq. (2) are defined as $U = \partial \Psi / \partial Z$, $W = -\partial \Psi / \partial X$.

The corresponding non-dimensional boundary conditions are

$$u(x, y, z) = -1 \text{ at } y = \pm 1,$$

 $\theta(x, y, z) = c_1 \text{ at } y = 1 \text{ and } \theta(x, y, z) = c_3 \text{ at } z = -1,$ (20)

$$\varphi(x, y, z) = c_2$$
 at $y = 1$ and $\varphi(x, y, z) = c_4$ at $z = -1$,

$$\begin{aligned} & u(x, y, z) = -1 \text{ at } z = \pm h(x) \\ & \theta(x, y, z) = 1 \text{ at } z = -h(x) \text{ and } \theta(x, y, z) = 0 \text{ at } z = h(x), \\ & \varphi(x, y, z) = 1 \text{ at } z = -h(x) \text{ and } \varphi(x, y, z) = 0 \text{ at } z = h(x), \end{aligned}$$
 (21)

where $0 \leqslant \phi \leqslant 1$. For straight duct $\phi = 0$ whereas $\phi = 1$ corresponds to total occlusion.

3. Solution of the problem

The analytical solutions of non-homogenous partial differential given in Eqs. (17) to (19) satisfying the boundary conditions (20) and (21) are

$$u(x,y,z) = -1 + \frac{1}{2} \frac{dp}{dx} (z^2 - h^2) + \sum_{m=1}^{\infty} A_m \cos \sqrt{\lambda_m} z \cosh \frac{\sqrt{\lambda_m}}{\beta} y,$$
(22)

$$\theta(x,y,z) = \sum_{m=1}^{\infty} \frac{C_m \cos \sqrt{\lambda_m} z}{\sec h \frac{\sqrt{\lambda_m}}{\beta} y} + \frac{D_m \cos \sqrt{\lambda_m} z}{\csc h \frac{\sqrt{\lambda_m}}{\beta} y} + \frac{1}{24h\lambda_m} \\ \left((12h\lambda_m + 2Eh^5 (dp/dx)^2 \Pr \lambda_m - 12z\lambda_m - 2Eh(dp/dx)^2 \Pr z^4 \lambda_m + 3A_m^2 Eh \Pr \lambda_m \cos 2h \sqrt{\lambda_m} - 3A_m^2 Eh \Pr \lambda_m \cos 2z \sqrt{\lambda_m} + 96A_m Eh(dp/dx) \Pr \cosh \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{\beta} y - 96A_m Eh(dp/dx) \\ \Pr \cos z \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{\beta} y + 6A_m^2 Eh^3 \Pr \lambda_m^2 (\cosh \frac{\sqrt{\lambda_m}}{\beta} y)^2 \\ -6A_m^2 Eh \Pr z^2 \lambda_m^2 \left(\cosh \frac{\sqrt{\lambda_m}}{\beta} y \right)^2 + 48A_m Eh^2 (dp/dx) \Pr \sqrt{\lambda_m} \\ \cosh \frac{\sqrt{\lambda_m}}{\beta} y \sin h \sqrt{\lambda_m} - 48A_m Eh(dp/dx) \Pr z \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{\beta} \\ \sin z \sqrt{\lambda_m} + 6A_m^2 Eh^3 \Pr \lambda_m^2 \left(\sinh \frac{\sqrt{\lambda_m}}{\beta} y \right)^2 - 6A_m^2 Eh \Pr \lambda_m^2 z^2 \left(\sinh \frac{\sqrt{\lambda_m}}{\beta} y \right)^2 \right),$$
(23)

$$\varphi(x,y,z) = \sum_{m=1}^{\infty} \frac{E_m}{\sec\sqrt{\lambda_m}z \sec h\frac{\sqrt{\lambda_m}}{\beta}y} + \frac{F_m \cos\sqrt{\lambda_m}z}{\csc h\frac{\sqrt{\lambda_m}}{\beta}y} + \frac{1}{24h\lambda_m} 12h\lambda_m$$

$$- 2Eh^5 (dp/dx)^2 \operatorname{PrScSr}\lambda_m - 12z\lambda_m - 2Eh(dp/dx)^2 ScSr \operatorname{Pr}z^4\lambda_m$$

$$- 3A_m^2 Eh \operatorname{Pr}\lambda_m ScSr \cos 2h\sqrt{\lambda_m} + 3A_m^2 Eh ScSr \operatorname{Pr}\lambda_m$$

$$\times \cos 2z\sqrt{\lambda_m} - 96A_m Eh(dp/dx) \operatorname{PrScSr} \cosh \frac{\sqrt{\lambda_m}}{\beta}y$$

$$\times \cos h\sqrt{\lambda_m} + 96A_m Eh(dp/dx) \operatorname{PrScSr} \cosh \frac{\sqrt{\lambda_m}}{\beta}y$$

$$\times \cos z\sqrt{\lambda_m} - 6A_m^2 Eh^3 \operatorname{PrScSr}\lambda_m^2 \left(\cosh \frac{\sqrt{\lambda_m}}{\beta}y\right)^2$$

$$+ 6A_m^2 Eh z^2 \operatorname{PrScSr}\lambda_m^2 \left(\cosh \frac{\sqrt{\lambda_m}}{\beta}y\right)^2$$

$$- 48A_m Eh^2 (dp/dx) \operatorname{PrScSr}\sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{\beta}y \sin h$$

$$\times \sqrt{\lambda_m} + 48A_m Eh(dp/dx) \operatorname{PrScSr}z\sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{\beta}y \sin h$$

$$\times \sqrt{\lambda_m} - 6A_m^2 Eh^3 \operatorname{PrScSr}\lambda_m^2 \left(\sinh \frac{\sqrt{\lambda_m}}{\beta}y\right)^2$$

$$+ 6A_m^2 Eh z^2 \operatorname{PrScSr}\lambda_m^2 \left(\sinh \frac{\sqrt{\lambda_m}}{\beta}y\right)^2$$

$$+ 6A_m^2 Eh z^2 \operatorname{PrScSr}\lambda_m^2 \left(\sinh \frac{\sqrt{\lambda_m}}{\beta}y\right)^2, \qquad (24)$$

where

$$\begin{split} A_{m} &= \sum_{m=1}^{\infty} \frac{1}{h \cos h \frac{\sqrt{\lambda_{m}}}{\beta}} \left(-2h \frac{dp}{dx} \frac{\cos h \sqrt{\lambda_{m}}}{\lambda_{m}} + 2 \frac{dp}{dx} \frac{\sin h \sqrt{\lambda_{m}}}{\lambda_{m}^{3/2}} \right), \\ C_{m} &= \sum_{m=1}^{\infty} \frac{1}{6h \lambda_{m}^{5/2}} \frac{1}{\cosh \frac{\sqrt{\lambda_{m}}}{\beta}} - 48Eh (dp/dx)^{2} \Pr \sqrt{\lambda_{m}} \cos \sqrt{\lambda_{m}} h \\ &+ 8Eh^{3} (dp/dx)^{2} \Pr \lambda_{m}^{3/2} \cos \sqrt{\lambda_{m}} h + 12A_{m}^{2} E \Pr \lambda_{m}^{2} \left(\cosh \frac{\sqrt{\lambda_{m}}}{\beta} y \right)^{2} \\ &\times (h \sqrt{\lambda_{m}} \cos \sqrt{\lambda_{m}} h - \sin \sqrt{\lambda_{m}} h) + 48Eh (dp/dx)^{2} \Pr \sin \sqrt{\lambda_{m}} h \\ &- 24Eh^{2} \lambda_{m} (dp/dx)^{2} \Pr \sin \sqrt{\lambda_{m}} h + 3A_{m}^{2} E \Pr \lambda_{m}^{2} \sin \sqrt{\lambda_{m}} h \\ &+ 6A_{m} E (dp/dx) \Pr \lambda_{m} \cos h \frac{\sqrt{\lambda_{m}}}{\beta} y (4h \sqrt{\lambda_{m}} + 2h \sqrt{\lambda_{m}} \cos 2\sqrt{\lambda_{m}} h \\ &- 3 \sin 2 \sqrt{\lambda_{m}} h) - A_{m}^{2} E \Pr \lambda_{m}^{2} \sin \sqrt{\lambda_{m}} h + 12A_{m}^{2} E \Pr \lambda_{m}^{2} \\ &\times \left(\sinh \frac{\sqrt{\lambda_{m}}}{\beta} y \right)^{2} (h \sqrt{\lambda_{m}} \cos \sqrt{\lambda_{m}} h - \sin \sqrt{\lambda_{m}} h), \end{split}$$

$$\begin{split} D_m &= \sum_{m=1}^{\infty} \frac{(c_1 - c_3) \sin \sqrt{\lambda_m} h \csc h \frac{\sqrt{\lambda_m}}{p}}{h \sqrt{\lambda_m}}, \\ E_m &= \sum_{m=1}^{\infty} -\frac{2 \sin \sqrt{\lambda_m} h}{h \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{p}} + \frac{2 c_2 \sin \sqrt{\lambda_m} h}{h \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{p}} + \frac{2 c_4 \sin \sqrt{\lambda_m} h}{h \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{p}} \\ &+ \frac{3 E h^3 (dp/dx)^2 \text{PrScSr} \sin \sqrt{\lambda_m} h}{3 h \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{p}} + A_m^2 \text{EPrScSr} \left(\cosh \frac{\sqrt{\lambda_m}}{\beta} y \right)^2 \\ &\times \frac{\sin \sqrt{\lambda_m} h}{2 h \sqrt{\lambda_m} \cosh \frac{\sqrt{\lambda_m}}{p}} + 8 A_m \text{EPrScSr} (dp/dx) \frac{\cosh \frac{\sqrt{\lambda_m}}{p} y (\sin \sqrt{\lambda_m} h)^2}{\lambda_m \cosh \frac{\sqrt{\lambda_m}}{p}} \\ &- \frac{1}{h \lambda_m} \frac{A_m^2 \text{EPrScSr}}{\cosh \frac{\sqrt{\lambda_m}}{p}} \left(\cosh \frac{\sqrt{\lambda_m}}{p} y \right)^2 (2 h \sqrt{\lambda_m} \cos \sqrt{\lambda_m} h) \\ &+ (h^2 \lambda_m - 2) \sin \sqrt{\lambda_m} h \right) - \frac{E (dp/dx)^2 \text{PrScSr}}{6 \cosh \frac{\sqrt{\lambda_m}}{p}} \\ &\times \left(\frac{8 h (h^2 \lambda_m - 6) \cos \sqrt{\lambda_m} h}{\lambda_m^{3/2}} + \frac{2 (24 - 12 h^2 \lambda_m + h^4 \lambda_m^2) \sin \sqrt{\lambda_m} h}{\lambda_m^{5/2}} \right) \\ &+ \frac{A_m \text{EPrScSr} (dp/dx)}{h \lambda_m^{3/2}} \cosh \frac{\sqrt{\lambda_m}}{p} y (2 h \sqrt{\lambda_m} \cos 2 \sqrt{\lambda_m} h - \sin 2 \sqrt{\lambda_m} h) \\ &+ \frac{A_m \text{EPrScSr} (dp/dx)}{h \lambda_m^{3/2}} \cosh \frac{\sqrt{\lambda_m}}{p} (3 \sin \sqrt{\lambda_m} h) \\ &+ \frac{A_m 4 E (dp/dx) \text{PrScSr}}{h 12 \sqrt{\lambda_m}} \cosh \frac{\sqrt{\lambda_m}}{p} (3 \sin \sqrt{\lambda_m} h + \sin 3 \sqrt{\lambda_m} h) \\ &+ A_m^2 \text{EhPrScSr} \sqrt{\lambda_m} \left(\frac{\sin \sqrt{\lambda_m} (\sin \frac{\sqrt{\lambda_m}}{p} y)^2}{\cosh \frac{\sqrt{\mu}}{p}} \right) \\ &- \frac{A_m^2 \text{EPrScSr}}{h \sqrt{\lambda_m}} (\sin \frac{\sqrt{\lambda_m}}{p} y)^2 (2 h \sqrt{\lambda_m} \cos \sqrt{\lambda_m} h + \sin 3 \sqrt{\lambda_m} h) \\ &+ A_m^2 \text{EhPrScSr} \sqrt{\lambda_m} \left(\frac{\sin \sqrt{\lambda_m} h (\sin \frac{\sqrt{\lambda_m}}{p} y)^2}{\cosh \frac{\sqrt{\mu}}{p}} \right) \\ &- \frac{A_m^2 \text{EPrScSr}}{h \sqrt{\lambda_m} \cos \frac{\sqrt{\mu}}{p}} (\sin \frac{\sqrt{\lambda_m}}{p} y)^2 (2 h \sqrt{\lambda_m} \cos \sqrt{\lambda_m} h + (h^2 \lambda_m - 2) \sin \sqrt{\lambda_m} h), \\ &+ (h^2 \lambda_m - 2) \sin \sqrt{\lambda_m} h), \\ &F_m = \sum_{m=1}^{\infty} \frac{(c_2 - c_4) \sin \sqrt{\lambda_m} h}{h \sqrt{\lambda_m} \sin \frac{\sqrt{\lambda_m}}{p}} . \end{split}$$

The volumetric flow rate q in a rectangular duct can be calculated as

$$q = \int_0^1 \int_0^{h(x)} u(x, y, z) \, dy dz.$$
(28)

The instantaneous flux is given by

$$\bar{Q} = \int_0^1 \int_0^{h(x)} (u+1) \, dy \, dz = q + h(x). \tag{29}$$

The average volume flow rate over one period $(T = \lambda/c)$ of the peristaltic wave is defined as



Fig. 22. Stream lines for different values of ϕ , (a) for $\phi = 0.3$, (b) for $\phi = 0.4$, (c) for $\phi = 0.5$, (d) for $\phi = 0.6$. The other parameters are $K^* = 0.4$, $\beta = 0.2$, Q = 0.01, y = 0.9.

$$Q = \frac{1}{T} \int_0^T \bar{Q} \, dt = q + 1. \tag{30}$$

The pressure gradient dp/dx is obtained after solving Eqs. (29) and (30).

$$\frac{dp}{dx} = (3(1 - h(x) - Q)\lambda_m^2(2h(x)\sqrt{\lambda_m} + \sin 2h(x)\sqrt{\lambda_m})) / (2h^4\lambda_m^{5/2} + h^3\lambda_m^2\sin 2h(x)\sqrt{\lambda_m} - 12\beta\sin^2 h(x) \times \sqrt{\lambda_m}\tanh\frac{\sqrt{\lambda_m}}{\beta} + 6h(x)\beta\sqrt{\lambda_m}\sin 2h(x)\sqrt{\lambda_m}\tanh\frac{\sqrt{\lambda_m}}{\beta}).$$
(31)

Also, the non-dimensional pressure rise Δp is evaluated numerically by using the following expression

$$\Delta p = \int_0^1 \frac{dp}{dx} \, dx. \tag{32}$$

Numerical integration for the integral given in Eq. (32) is performed using software built-in Mathematica.

4. Results and discussion

In fact the aim of this section is in three folds. Firstly the behavior of parameters involved in the expressions of velocity *u*, temperature θ and mass concentration φ . Secondly and thirdly the pumping characteristics and trapping mechanism respectively are taken into account. For this purpose Figs. 2–24 are sketched to measure the features of all parameters. In particular, the variations of Reynolds number, Prandtl number, Eckert number, Schmidt and Soret numbers are examined.

Figs. 2 and 3 illustrate the variations of velocity *u*. Fig. 2 shows that by increasing flow rate Q, velocity filed increases. From Fig. 3 it can be seen that with increase of aspect ratio β , the velocity field



Fig. 23. Stream lines for different values of Q_{c} (a) for Q = 0.01, (b) for Q = 0.03, (c) for Q = 0.07, (d) for Q = 0.1. The other parameters are $K^* = 0.4$, $\beta = 0.1$, $\phi = 0.45$, y = 0.9.

decreases and maximum velocity is obtained in the centre of channel. Figs. 4–7 describe the variation of temperature profile. From Fig. 4, it can be observed that with the increase of flow rate Q, temperature profile increases. It can also noticed from Fig. 5 that temperature profile decreases by decreasing the Prandtl Number Pr. Fig. 6 shows that with the increase of Eckert number E, the temperature profile increases. Fig. 7 illustrates the effects of aspect ratio β on temperature profile. From this figure it can be observed that temperature near the walls has same for all values of aspect ratio β but in the region $z \in [-0.5, 0.5]$, the temperature profile increases by increasing the value of β . Figs. 8–13 are plotted to study the behavior of Eckert number E, Schmidt number Sc, Soret number Sr, flow rate Q, aspect ratio β and Prandtl number Pr on concentrated profile φ . Concentration distribution for different values of β is shown in Fig. 8. It is depicted that concentration decreases with the increase of β . It also reveals from Fig. 9 that concentration profile decreases by increasing the Soret number *Sr.* Figs. (10) and (11) describe the simultaneous effects of Schmidt number *Sc* and flow rate *Q* on concentration. It is seen that concentration decreases with the increase of Schmidt number *Sc* and flow rate *Q*. Fig. 12 displays against the various values of Prandtl number Pr. It is depicted that concentration profile is minimum for the small values of Pr. Effects of Eckert number on concentration are shown in Fig. 13. It is observed that the concentration profile decreases with the increase of Eckert number.

Figs. 14–17 illustrates the variation of dp/dx versus x. It reveals from Fig. 14 with the increase of aspect ratio β in the narrow part of the channel $x \in [0.3, 0.7]$, the pressure gradient increases while in the wider parts of the channel $x \in [0, 0.3]$ and $x \in [0.7, 1]$, the pressure gradient is decreasing. Fig. 15 depicts that the pressure gradient decreases for large values of K^* . Fig. 16 shows that when amplitude ratio ϕ , increases the pressure gradient increases in



Fig. 24. Stream lines for different values of K^* , (a) for $K^* = 0.2$, (b) for $K^* = 0.3$, (c) for $K^* = 0.4$, (d) for $K^* = 0.5$. The other parameters are $\beta = 0.2$, $\phi = 0.4$, Q = 0.01, y = 0.9.

Table	1
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The convergence of analytical series solutions given in Eq. (23) is achieved at 10th-order of approximations, that is $A_m \rightarrow 0$ as $m \rightarrow \infty$.

$m ightarrow \infty$	$A_m ightarrow 0$
1	-1.52443
2	0.00113
3	0.00001
4	1.87871×10^{-7}
5	4.06248×10^{-9}
6	1.02262×10^{-10}
7	2.84733×10^{-12}
8	8.51862×10^{-14}
9	$2.6895 imes 10^{-15}$
10	8.85389×10^{-17}



Fig. 25. Velocity profile for rectangular duct $(\beta \neq 0)$ and rectangular channel $(\beta = 0)$.



Fig. 26. Concentration profile for rectangular duct $(\beta \neq 0)$ and rectangular channel $(\beta = 0)$.



Fig. 27. Temperature profile for rectangular duct $(\beta \neq 0)$ and rectangular channel $(\beta = 0)$.

Table 2 Difference between the rectangular duct $(\beta \neq 0)$ and the rectangular channel $(\beta = 0)$.

h(x)	u(x, y, z) $\beta = 0$	u(x, y, z) $\beta \neq 0$	$egin{aligned} & \varphi(\mathbf{x},\mathbf{y},\mathbf{z}) \ & \beta = 0 \end{aligned}$	$ \begin{aligned} \varphi(\mathbf{x},\mathbf{y},\mathbf{z}) \\ \beta \neq 0 \end{aligned} $	$\theta(x, y, z)$ $\beta = 0$	$\theta(x, y, z)$ $\beta \neq 0$
$-0.85 \\ -0.68 \\ -0.51 \\ -0.34 \\ -0.17 \\ 0 \\ 0.17$	-1 -0.4600 -0.0399 0.2600 0.4400 0.5000 0.4400	-1 -0.5232 -0.1731 0.0650 0.2029 0.2480 0.2020	1 0.4572 0.1471 -0.0308 -0.1488 -0.2500 0.2488	1 0.2820 -0.1094 -0.3164 -0.4401 -0.5413 0.6401	1 1.3428 1.4528 1.4308 1.3488 1.2500 1.1488	1 1.5167 1.7072 1.7134 1.6366 1.5376
0.17 0.34 0.51 0.68 0.85	0.4400 0.2600 -0.0399 -0.4600 -1	0.2029 0.0650 -0.1731 -0.5232 -1	-0.3488 -0.4308 -0.4528 -0.3428 0	-0.6401 -0.7164 -0.7094 -0.5179 0	1.1488 1.0308 0.8528 0.5428 0	1.4366 1.3134 1.1072 0.7167 0

narrow part of the channel whereas decreasing behavior is observed in the wider parts of the channel.

The effects pressure gradient versus flow rate are depicted in Fig. 17. It shows that flow rate decreases by an increase in the pressure gradient.

Figs. 18–20 present the variations of pressure rise Δp against the flow rate *Q*. Fig. 18 depicts that in retrograde pumping $(\Delta p > 0, Q < 0)$ region, the pumping rate increases by the increase of aspect ratio β but gives opposite behavior in copumping $(\Delta p < 0, Q > 0)$ region. It can be noticed from Fig. 19 that when amplitude ratio ϕ increases then in retrograde pumping $(\Delta p > 0, Q < 0)$ region the pumping rate increases while in copumping region $(\Delta p < 0, Q > 0)$ when $Q \in [0, 0.4]$ its behavior remain same whereas in copumping region when $Q \in [0.4, 2]$ attitude of Δp is quite opposite. Influence of non uniform parameter *K*^{*} is shown in Fig. 20. It is evident from results that an increase in *K*^{*} pumping rate increases in retrograde pumping and copumping regions $(\Delta p > 0, Q < 0)$ and $(\Delta p < 0, Q > 0)$ respectively

when $Q \in [0, 0.5]$. An opposite effect is seen in copumping $(\Delta p < 0, Q > 0)$ region when $Q \in [0.5, 2]$. In peristaltic motion trapping is another interesting phenomena. Basically it is formulation of an internally circulating bolus of fluid by closed stream lines. The trapped bolus pushed a head along a peristaltic waves. In the wave frame, streamlines under certain conditions split to trap a bolus which moves as a whole with the speed of the wave. The formation of an internally circulating bolus of the fluid by closed streamline is called trapping. The bolus defined as a volume of fluid bounded by a closed streamlines in the wave frame is transported at the wave. For this purpose, the trapping phenomenon is presented by plotting stream lines and are shown in Figs. 21-24 for various parameters. The stream lines for various values of β is sketched in Fig. 21. It is depicted that with the rise of aspect ratio β , the size of trapping bolus decreases but the size of bolus increases. Fig. 22 displays for amplitude ration ϕ , it is noted that the size of the bolus becomes smaller and trapping bolus increases with the increase of ϕ . The stream lines for different values of flow rate Q are sketched in Fig. 3, It is seen that the trapping bolus reduces with an increase of Q. Fig. 24, illustrates the effects of K^* . It is measured here that when the parameter K increases then the size of bolus increases and as a result trapping bolus reduce in numbers.

Moreover, one can easily observe that the convergence of analytical series solutions given in Eq. (23) depends on A_m . It is found that the convergence of series solutions is achieved at 10th-order of approximations (Table 1). In Figs. 25–27, we present the effects of rectangular channel ($\beta = 0$) and rectangular duct ($\beta \neq 0$) for non uniform channel on velocity, concentration and temperature profiles. It is found that the velocity and concentration layers in case of rectangular duct ($\beta \neq 0$) are smaller than rectangular channel ($\beta = 0$). However, the temperature layer in rectangular duct is larger than the rectangular channel. Furthermore, the results for rectangular channel can be recovered by taking $\beta = 0$. It is noticed that when $\beta = 1$ the rectangular duct becomes a square duct. Finally a comparison is also given in Table 2 in order to show the difference of presented study with the existing literature.

5. Concluding remarks

In this paper, peristaltic flow in a non-uniform rectangular duct has been studied. The influence of heat transfer is taken into account. Analysis for Prandtl Number Pr, Eckert number *E*, Schmidt number *Sc* and Soret number *Sr* is presented. The features of the flow characteristics are analyzed in detail by plotting graphs and numerical tables. It is found that as $m \to \infty \Rightarrow A_m \to 0$ that is the convergence of the solutions is achieved at 10th-order of approximations. Finally a comparison with the existing literature is also made. It is observed that the results for rectangular channel can be recovered by taking $\beta = 0$. It is also noticed that when $\beta = 1$ the rectangular duct becomes a square duct. The results obtained for the flow of peristaltic fluid in a non uniform duct reveal many interesting behaviors that warrant further study on the non-Newtonian fluid phenomena, especially the shear-thinning phenomena. Shear-thinning reduces the wall shear stress.

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