Closure to "Analysis of Asymmetric Disk-Shaped and Flat-Plate Heat Pipes"

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We would like to thank Kim et al. for appreciating the importance of our work [1]. It should be noted that the introduction of a disk shaped heat pipe by us in the cited paper was done for the very first time in the literature and our comprehensive analytical results for the disk-shaped and rectangular-shaped heat pipes constitute the very first such analysis in the literature. In summary, we are going to show that all of our derivations and results are exactly correct as presented in the paper. We only have a couple of minor misprints in the paper that need to be cited. However, all of our presented analytical results and figures are correct as they are. We will rigorously show that the points raised by Kim et al. are partly due to their misunderstanding of our analysis and partly due to the errors in their work. In fact, we show that our work is indeed very much correct. In what follows, we will show that each and every one of the points brought up by Kim et al. is either a nonissue or a mistake. In summary, Kim et al. claim that

- For the vapor pressure gradient equation, Kim et al. wrongly claim that our Eq. (16) [2] is incorrect as it is missing a term $\left([1/3]\left[1/\mathrm{Re}_{\mathrm{h}}^2 r^+\right]\right)$, They then provided Eq. (1) in their discussion statement which they claim is the correct equation. Other than a typo or mistake that a Re_{h} is missing in the first term of their equation for $0 \le r^+ \le \varphi R^+$, the only difference between their Eq. (1) and author's Eq. (16) is that they replaced the term $\left(\left[4/\Phi^2\right]\left[1/\mathrm{Re}_{\mathrm{h}}^2 r^+\right]\right)$ in the author's Eq. (16) with the term $\left([1/3]+\left[4/\Phi^2\right]\right)\left(1/\mathrm{Re}_{\mathrm{h}}^2 r^+\right)$. We will rigorously show that their presentation of this revised term is without a physical basis and incorrect.
- For the nondimensionalized liquid pressure drop equation, Kim et al. claim that, instead of $p_{\mathrm{l}}^+(R^+) = p_{\mathrm{v}}^+(R^+)$, the boundary condition at $r^+ = R^+$ should be $p_{\mathrm{l}}^+(R^+) = (1/\rho^+)p_{\mathrm{v}}^+(R^+)$, which leads to their Eq. (3) for the simplified analytical solution of liquid pressure drop. They also presented their calculated liquid pressure drop and maximum input power using their Eq. (3) to show a discrepancy between their correct results and the author's results (Figs. 1 and 2 in their statement). However, their statement is incorrect as we show it rigorously below.

In what follows, we rigorously show that their claims have no basis and their mistakes are partly due to their misunderstanding of our analysis and partly due to their errors in their work. We also point out the two minor misprints in our results.

1 Vapor Pressure Gradient Equation

The vapor pressure gradient equation was obtained by integrating the r^+-momentum equation, Eq. (2) in our work (the cited paper), within a channel bounded by porous wicks. As stated very clearly in our paper (page 212), "Since the dimension in the r^+ direction is much larger than the transverse length in the vapor channel, the shear stress in the r^+ direction will be neglected [3,4]. We have also been able to confirm that the shear stress in the r^+ direction is negligible when it was accounted for in the analysis." By neglecting the shear stress in the r^+ direction, which is $\left[\partial^2 u_{\mathrm{v}}^+/\partial(r^+)^2\right] + \left[1/r^+\right]\left[\partial u_{\mathrm{v}}^+/\partial r^+\right] - \left[u_{\mathrm{v}}^+/(r^+)^2\right]$, we were able to derive Eq. (16) in our paper (the cited paper) for the vapor pressure gradient equation.

If, instead of neglecting the entire r^+ direction shear stress $\left[\partial^2 u_{\mathrm{v}}^+/\partial(r^+)^2\right] + \left[1/r^+\right]\left[\partial u_{\mathrm{v}}^+/\partial r^+\right] - \left[u_{\mathrm{v}}^+/(r^+)^2\right]$, only neglecting $\left[\partial^2 u_{\mathrm{v}}^+/\partial(r^+)^2\right] + \left[1/r^+\right]\left[\partial u_{\mathrm{v}}^+/\partial r^+\right]$ but keeping the $-\left[u_{\mathrm{v}}^+/(r^+)^2\right]$ term in the vapor r^+-momentum equation, the integration of the vapor r^+-momentum equation results in Eq. (1) presented by Kim et al. in their discussion of our paper. Therefore, we have definitive reason to believe that Kim et al. kept the $-\left[u_{\mathrm{v}}^+/(r^+)^2\right]$ term during their integration of the momentum equation, i.e., they neglected part of the r-direction shear stress and kept part of it. This does not make sense physically. This points to a misunderstanding of the underlying physics of the analysis.

2 Liquid Pressure Equation

The liquid pressure equation was derived as the nondimensional liquid pressure which is defined as $p_{\mathrm{l}}^+ = p_{\mathrm{l}}/\rho_{\mathrm{v}} \mathrm{v}_1^2$. There is a typo in the definition of p_{l}^+ in the nomenclature section of our paper. Instead of the vapor density, the liquid density was mistakenly printed in the nomenclature. We believe this is the reason that Kim et al. claim that the correct boundary condition should be $p_{\mathrm{l}}^+(R^+) = (1/\rho^+)p_{\mathrm{v}}^+(R^+)$ and the correct liquid pressure equation should be Eq. (3) in their statement. However, the only difference is just due to this typo and the results are correct as presented. As such, we will clarify the following points.

(a) There is a typo in the nondimensional liquid pressure given in the nomenclature of our paper. The definition $p_{\mathrm{l}}^+ = p_{\mathrm{l}}/\rho_{\mathrm{v}} \mathrm{v}_1^2$ was used throughout our analytical work. This can be verified from another one of our related papers [5] on page 165.

(b) The boundary condition $p_{\mathrm{l}}^+(R^+) = p_{\mathrm{v}}^+(R^+)$ is correct as well as the liquid pressure equation (Eq. (33) in our cited paper). It should just noted that $p_{\mathrm{l}}^+ = p_{\mathrm{l}}/\rho_{\mathrm{v}} \mathrm{v}_1^2$ is what we have used in those equations.

(c) We have recast the liquid pressure equation using $p_{\mathrm{l}}^+ = p_{\mathrm{l}}/\rho_{\mathrm{l}} \mathrm{v}_1^2$ and obtained the following equation:

$$p_{\mathrm{l}}^+(r^+) = \begin{cases} \dfrac{1}{\rho^+} p_{\mathrm{v}}^+(0) + \dfrac{\nu^+ \mathrm{Re}_{\mathrm{h}} (R^+)^2}{4\rho^+ K^+ \left(h_{\mathrm{w}}^+\right)^3} \dfrac{1-\varphi^2}{2-\varphi^2} \\ \quad \times \left[\left(\dfrac{r^+}{R^+}\right)^2 + \dfrac{2\varphi^2}{1-\varphi^2}\ln\varphi\right] \\ \quad - \dfrac{1}{\rho^+}\left(\dfrac{4}{5}\dfrac{\varphi^2 \mathrm{Re}_{\mathrm{h}}}{2-\varphi^2} R^+\right)^2 \\ \quad \times \left(2\ln\varphi + \dfrac{1}{\varphi^2} - 1\right), \quad (0 \le r^+ \le \varphi R^+) \\ \dfrac{1}{\rho^+} p_{\mathrm{v}}^+(0) + \dfrac{\nu^+ \mathrm{Re}_h (R^+)^2}{4\rho^+ K^+ \left(h_{\mathrm{w}}^+\right)^3} \dfrac{\varphi^2}{2-\varphi^2} \\ \quad \times \left[1 - \left(\dfrac{r^+}{R^+}\right)^2 - 2\ln\left(\dfrac{R^+}{r^+}\right)\right] \\ \quad - \dfrac{1}{\rho^+}\left(\dfrac{4}{5}\dfrac{\varphi^2 \mathrm{Re}_{\mathrm{h}}}{2-\varphi^2} R^+\right)^2 \\ \quad \times \left(2\ln\varphi + \dfrac{1}{\varphi^2} - 1\right), \quad (\varphi R^+ \le r^+ \le 1) \end{cases} \tag{1}$$

Except for the factor $(1/\rho^+)$, which is missing (which is an error in their equation) in the $p_{\mathrm{v}}^+(0)$ term of Eq. (3) that Kim et al. presented in their discussion statement, the rest of our Eq. (1)

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Contributed by the Heat Transfer Division of ASME for publication in the Journal of Heat Transfer. Manuscript received July 30, 2014; final manuscript received August 21, 2014; published online September 16, 2014. Assoc. Editor: Bruce L. Drolen.

given above is same as Eq. (3) presented by Kim et al. in their discussion statement.

(d) It should be noted that the calculated vapor and liquid pressure drop in Fig. 4 of our work is presented in dimensional form. First, we converted the nondimensional liquid pressure equation, Eq. (33) in our paper to dimensional liquid pressure as given below, using $p_l^+ = p_l/\rho_v v_1^2$

$$p_l(r^+) = \begin{cases} p_v(0) + \dfrac{\mu_v v_l \mathrm{Re}_h^3 (R^+)^2}{4K h_w^+} \dfrac{1-\varphi^2}{2-\varphi^2} \\ \quad \times \left[\left(\dfrac{r^+}{R^+} \right)^2 + \dfrac{2\varphi^2}{1-\varphi^2} \ln \varphi \right] \\ \quad - \dfrac{\mu_v^2}{\rho_v h^2} \mathrm{Re}_h^4 \left(\dfrac{4}{5} \dfrac{\varphi^2}{2-\varphi^2} R^+ \right)^2 \\ \quad \left(2 \ln \varphi + \dfrac{1}{\varphi^2} - 1 \right), \quad (0 \le r^+ \le \varphi R^+) \\ p_v(0) + \dfrac{\mu_v v_l \mathrm{Re}_h^3 (R^+)^2}{4K h_w^+} \dfrac{\varphi^2}{2-\varphi^2} \\ \quad \times \left[1 - \left(\dfrac{r^+}{R^+} \right)^2 - 2 \ln \left(\dfrac{R^+}{r^+} \right) \right] \\ \quad - \dfrac{\mu_v^2}{\rho_v h^2} \mathrm{Re}_h^4 \left(\dfrac{4}{5} \dfrac{\varphi^2}{2-\varphi^2} R^+ \right)^2 \\ \quad \times \left(2 \ln \varphi + \dfrac{1}{\varphi^2} - 1 \right), \quad (\varphi R^+ \le r^+ \le 1) \end{cases} \tag{2}$$

We then converted Eq. (1), which is the same as Eq. (3) of Kim et al.'s discussion (other than their error in missing the factor $(1/\rho^+)$ in the $p_v^+(0)$ term of their Eq. (3)) to dimensional liquid pressure using $p_l^+ = p_l/\rho_l v_1^2$ and obtained the exact same equation as Eq. (2) above. This means that, other than the missing factor as noted above, the nondimensional liquid equation presented by Kim et al. (Eq. (3) in their statement) leads to same equation that we had developed for the dimensional liquid pressure.

Another point is that our results in Figs. 4 and 7 presented in our paper, which Kim et al. have used for comparison with their results, are given for $K = 1.5 \times 10^{-8}$ m^2. This value was not explicitly stated in our original paper [2]. The permeability that Kim et al. have used for their results, $K = 1.8 \times 10^{-9}$ m^2, is noted in an earlier portion (a misprint) of our paper [2] but was not the value used for the results in Figs. 4 and 7. If we use the same K value as used by Kim et al., we will get exactly the same results and figures (accounting for the missing factor $(1/\rho^+)$ in the $p_v^+(0)$ term which is an error in their equation). Conversely, if Kim et al. use the same K value that we had used they would obtain exactly the same results as we had obtained. We have calculated the radial liquid pressure distribution using our simplified equation and the

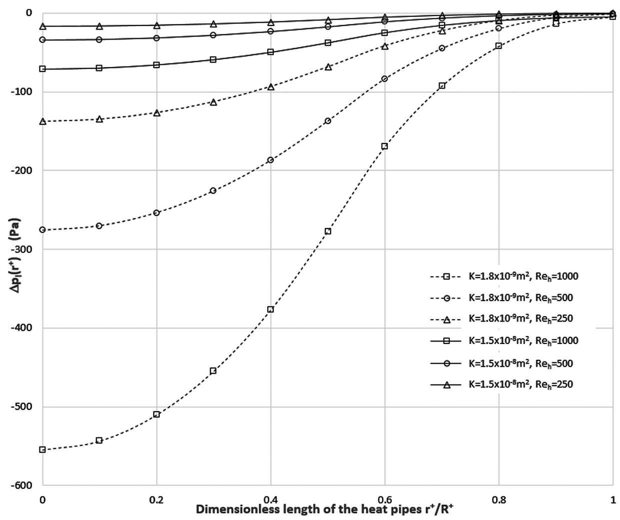

Fig. 1 Radial liquid pressure distribution using our simplified derived equation at different values of Reynolds numbers and permeabilities

results are presented in Fig. 1 at different values of Reynolds numbers and permeabilities. This figure very clearly shows that if Kim et al. had used the same permeability value that we had used, i.e., $K = 1.5 \times 10^{-8}$ m^2, in generating the liquid pressure distribution, they would have obtained precisely our results and conversely, if we had used the permeability that they had used, i.e., $K = 1.8 \times 10^{-9}$ m^2 their presented results would be obtained. It is clear that if they had utilized the pressure distribution for $K = 1.5 \times 10^{-8}$ m^2 that we had used they would have obtained precisely the same maximum heat flux values given in Fig. 7 of our paper [2]. As such, for brevity we will not present a figure to show this.

In summary, all of our derived equations and results are precisely correct as they are. We have rigorously shown that Kim et al. statements are entirely due to their misunderstanding of our analysis and errors in their results. However, we are thankful for their discussion so that we can point out a couple of minor misprints in our work.

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