STATISTICAL BOUNDS FOR THE EFFECTIVE THERMAL CONDUCTIVITY OF MICROSPHERE AND FIBROUS INSULATION

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Abstract

The present work consists of an analytical investigation of the effective thermal conductivity of microsphere and fibrous insulation media. Because of the statistical nature of random packing patterns of microspheres and fibers, there exists a range of effective thermal conductivities for any given solid fraction of the microsphere or fibrous insulation medium. An analysis that applies two standard variational principles to some model media simulating the microsphere and fibrous insulation has been carried out to determine the statistical upper and lower bounds of the effective thermal conductivity. The physical models composed of cells with different packing configurations in case of microspheres and special fiber-matrix arrangements in case of fibrous insulation significantly simplify the evaluation of the statistical bounds. In general, the pertinent parameters for determining the effective conductivity are thermal conductivities of the constituent phases, the volume fraction, and a constant that is a function of the cell geometry. Furthermore, contemplating the empirical results that particular types of packings occur more than others in the microsphere case, a computer simulation imparting this effect into determining the effective bounds has been devised. The present microsphere results are in excellent agreement with the previous analytical work using a regular-cell approach.

Nomenclature

E	=	Young's modulus of the solid sphere
F_1, F_2	=	functions defined in Eqs. (5) and (6)

Presented as Paper 78-874 at the Second AIAA/ASME Thermophysics and Heat Transfer Conference, Palo Alto, Calif., May 24-26, 1978.

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G	=	cell geometric factor
k	=	thermal conductivity
k_e	=	effective thermal conductivity
k_e^*	=	dimensionless effective thermal conductivity defined in Eq. (10)
$k_{e,a}$	=	axial effective thermal conductivity of fibrous insulation
$k_{e,t}$	=	transverse effective thermal conductivity of fibrous insulation
k_j	=	effective thermal conductivity of a unit cell structure of the jth kind
k_o	=	gas conductivity under continuum conditions at STP
k_s	=	thermal conductivity of the solid sphere
k_{ue}^*	=	dimensionless effective thermal conductivity defined in Eq. (13)
k_1	=	larger of the two constituent thermal conductivities
k_2	=	smaller of the two constituent thermal conductivities
K_j	=	thermal conductivity parameter for cells of the jth kind
P	=	loading pressure
S_j	=	solid fraction parameter for cells of the jth kind
α	=	ratio defined by k_1/k_2 ($\alpha \geqslant 1$)
δ_s	=	solid fraction of the material with property k_1
δ_v	=	volume fraction of the material with property k_1
μ	=	Poisson's ratio of the solid sphere

Introduction

Microsphere insulation, which represents a special type of packed beds, is basically a stagnant agglomerate of small unconsolidated spherical particles.[1] These beds, possessing a large ratio of solid surface area to their volume, provide excellent thermal insulation characteristics for a variety of industrial applications.[2,3] Determination of the thermal characteristics of such beds, especially the conduction contribution, long has been the subject matter of numerous studies.[4-6] However, previous works have dealt mainly with the regular arrangement of solid particles throughout the bed, thus implementing the idea that the thermal conductivity of a packed bed is a unique function of the solid fraction. That this is not the actual case was indicated first by Nayak and Tien, showing a range of thermal conductivities at each solid fraction.[7] Nevertheless, the random nature of packings in the bed was not taken into account. The present work accommodates the random arrangement of solid particles by assuming the packed bed to be composed of four different packing structures, randomly dispersed throughout the bed. Furthermore, the probability of occurrence of a certain packing over another has been incorporated into the analysis by a computer simulation.

Another widely employed insulation system is the fibrous insulation.[2,3] Previous works on the effective thermal conductivity of the fibrous insulation have been mostly empirical, but a few analytical studies have been made on fiber-reinforced materials.[8,9] The present work, which extends these previous studies, considers the fibers suspended in a continuum matrix of gas or solid phase. By devising two different models for the arrangement of fibers in the matrix, the resulting bounds on the effective thermal conductivity at different concentrations of fibers are presented for some typical values of the ratio of fiber to the interstitial-phase conductivity.

Theoretical Basis

The most fundamental bounds for the effective thermal conductivity of multiphase media are based directly on the information about their volume fractions and properties of constituent phases.[10] These bounds are acquired by assuming the series connection of the constituents as the lower bound and their parallel connection as the upper bound. Through such an analysis, the effective thermal conductivity k_e of a heterogeneous material with spatially varying thermal conductivity k falls into the range

$$\langle 1/k \rangle^{-1} \leqslant k_e \leqslant \langle k \rangle \tag{1}$$

where < > denotes spatial average of any quantity. However, this range is relatively wide, and the model, which will be called the volume fraction model, neglects any pertinent packing information other than the volume fractions. Therefore, an improved model carrying more information and thus providing a set of narrower bounds is needed.

By using two standard variational principles and applying the perturbation series expansion of heat flux and temperature gradient as trial functions into these principles, statistical bounds can be established in a more detailed manner for the effective thermal conductivity of a statistically homogeneous and isotropic multiphase medium.[11,12] These bounds are given as

$$k_e \leqslant \langle k \rangle - \frac{\langle k'^2 \rangle / 3 \langle k \rangle}{1 + (3 \langle k \rangle J_1 / \langle k'^2 \rangle)} \tag{2}$$

$$k_e \geqslant \left\{ < \frac{1}{k} > - \frac{[(4 < k'/k >/3)^2/4 < k >^2]}{[< k >^2 < k'^2/k >/3 + J_2]} \right\}^{-1} \qquad (3)$$

where the quantities J_1 and J_2 are associated with the spatial integrations of $< k'(0)k'(\vec{r})k'(\vec{s}) >$ and $< k'(\vec{r})k'(\vec{s})/k'(0) >$, respectively, $k' = k - < k >$, or deviations of k from the spatial average, and $\vec{r}$ and $\vec{s}$ denote positions in the vector space. The quantity $< k'(0)k'(\vec{r})k'(\vec{s}) >$ or $< k'(r)k'(s)/k'(0) >$ represents the so-called three-point correlation function, which is related basically to the summation of terms representing the probabilities of three points being in different combinations of the two-phase material. As is clear from the preceding results, the success of this method is primarily dependent upon the evaluation of J_1 and J_2, which, in turn, relies on the success in determining these three-point correlation functions.

A close examination of the two insulation media under consideration leads to an important conceptual model. In the case of microspheres, the bed can be considered to consist of cubes of a certain size distribution, so that the whole bed is covered with them. Each of these cubes is representative of a certain type of packing (e.g., simple cubic, body-centered cubic, etc.). In the case of fibrous insulation, the medium consists of needle-shaped fibers suspended in gaseous or solid matrix, which, in turn, can be regarded to consist of needles of gas or solid, both of which are of a certain size distribution so that they will cover the whole fibrous medium. The significance of this model is evident. In both cases, the media can be assumed to consist of cells, each of which has a material property independent of any other cell. This cell concept results in a remarkable simplification in the analysis by following an approach devised by Miller[13] in evaluating these three-point correlation functions. Instead of summation over quantities that are related to the probability of three points being in different combinations of the phases, summation now is over terms that are related to the probability of three points in different cells, thus creating some specified simple boundaries.

By pursuing the foregoing procedure, the spatial integrals of the correlation functions are cast in terms of a new factor, which is purely dependent upon the geometry of the cells, and the evaluation of which is quite simple for the cell geometries in the two cases considered here. Therefore, after adopting the foregoing procedure into the analysis, the following bounds can be established for the three-dimensional cell-type models:

$$F_1(\delta_v,\alpha,G) \geqslant (k_e/\sqrt{k_1 k_2}) \geqslant F_2(\delta_v,\alpha,G) \quad (4)$$

$$F_1 \equiv \frac{1+\delta_v(\alpha-1)}{\sqrt{\alpha}}$$

$$\left\{1 - \frac{\delta_v(\alpha-1)^2\,(1-\delta_v)}{3[1+\delta_v(\alpha-1)][1+\delta_v(\alpha-1)+3(\alpha-1)(1-2\,\delta_v)G]}\right\} \quad (5)$$

$$F_2 \equiv \left\{\sqrt{\alpha}\left\{(\alpha-\delta_v(\alpha-1) - \frac{[4(\alpha-1)^2(1-\delta_v)\delta_v]}{3[1+\alpha+3(2\,\delta_v-1)(\alpha-1)G]}\right\}^{-1} \quad (6)$$

where δ_v is the volume fraction of the material with property k_1, $\alpha = k_1/k_2$ with $k_1 > k_2$ or $\alpha > 1$, and G is the cell geometry of the cells. For three-dimensional geometries, G is 1/9 for spherical cells and 1/3 for platelike cells, representing two limiting cases.

Microsphere Insulation

Evacuated Packed Bed of Microspheres

The conductance of a packed bed is dependent on the packing pattern and the applied load. When considering spheres of uniform size, the three basic packings are body-centered cubic (bcc), face-centered cubic (fcc), and simple cubic (sc). These three packing structures, along with another type of packing, namely, simple cubic with a defect in the structure (scp) representing the structural defects in packing behavior, have been considered in the analysis so as to provide some convenient physical modeling for the conduction phenomena in the packed bed. The thermal conductivities of these basic unit-cell packing structures are given by Chan and Tien[4] as

$$k_j = K_j\,k_s[(1-\mu^2)P/E]^{1/3} \quad ; \; j = 1,2,\ldots \quad (7)$$

where K_j is a thermal conductivity parameter characteristic of different packing structures, P is the loading pressure, and μ and E are Poisson's ratio and Young's modulus of the solid sphere, respectively. Table 1 gives the values of K_j and S_j (a solid fraction parameter to be introduced later) for the four basic packing structures.

Table 1 Values of the thermal conductivity and solid fraction parameters for various types of packing

Packing type	K_j	S_j
Body-centered cubic	1.96	0.68
Face-centered cubic	2.89	0.74
Simple cubic	1.36	0.524
Simple cubic with a structural defect	1.21	0.393

Previous analyses on microspheres[4,5] have been based on the assumption that the packed bed is filled with just one type of unit-cell packing, which is not very realistic. In contrast, the present work considers the packed bed to be a mixture of two different packing structures. Two different approaches are pursued. The first approach is to consider the bed to consist of only two specific types of packings and then to find the upper and lower bounds for all possible pair mixtures of the four basic unit cells considered. The required thermal conductivities of different types of packings are obtained from Eq. (7). By designating the two different packing conductivities as k_1 and k_2, there follows

$$\sqrt{k_1 k_2} = \sqrt{K_1 K_2}\ k_s\ [(1-\mu^2)P/E]^{1/3} \tag{8}$$

Substituting Eq. (8) into Eq. (4) gives the upper and lower bounds for the effective thermal conductivity of a packed bed of microspheres:

$$\sqrt{K_1 K_2}\ F_1(\delta_v,\alpha,G) \geqslant k_e^* \geqslant \sqrt{K_1 K_2}\ F_2(\delta_v,\alpha,G) \tag{9}$$

where k_e^* is the dimensionless effective thermal conductivity defined by

$$k_e^* = k_e/k_s\ [(1-\mu^2)P/E]^{1/3} \tag{10}$$

For the three-dimensional cubic cells under consideration, G = 1/3. In many instances, the solid fraction δ_s is specified instead of δ_v, but they are related as follows:

$$\delta_s = \delta_v\ S_1 + (1-\delta_v)S_2 \tag{11}$$

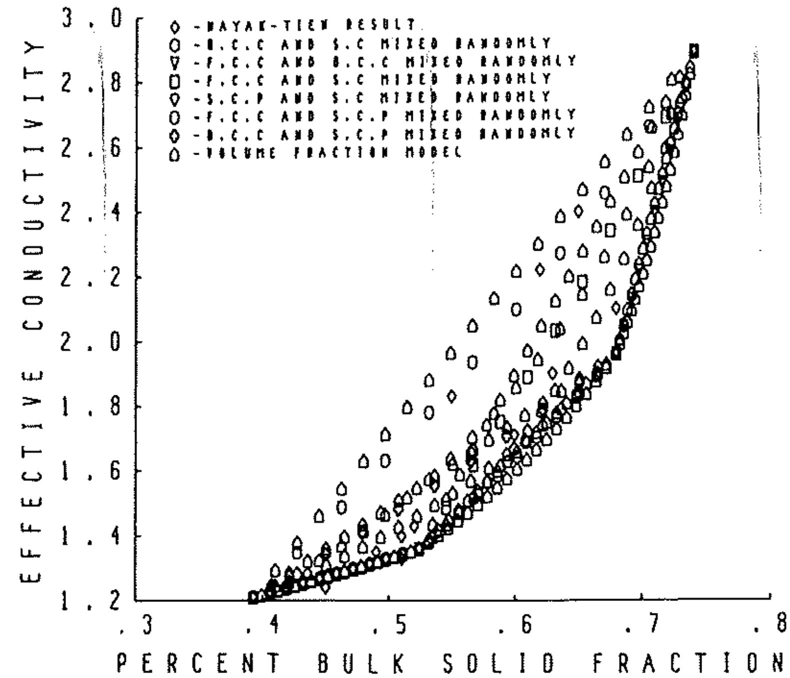

Fig. 1 Effective thermal conductivity of packed microspheres.

where S_1 and S_2 are the solid fraction parameters as given in Table 1. Figure 1 presents the results of the dimensionless effective thermal conductivity as a function of the solid fraction by mixing randomly any two of the four basic packing cells. Also shown in the figure are those points obtained from the simple volume fraction model as well as those from Nayak and Tien's regular-cell analysis.[7]

The second approach, which brings in the main feature of the analysis, is accomplished by incorporating the probability of finding a certain kind of packing over another in the packed bed of microspheres. By utilizing the experimental results of Bernal and Mason,[14] which give the probability of occurrence of different packings in a packed bed, a computer program is set up for incorporation of this probability distribution. At each step, two different unit-cell structures are chosen according to this probability distribution. Then it is assumed that the bed is a random mixture of the chosen packings, so that the bounds on the effective conductivity are obtained at the corresponding solid fraction. By numerous repetitions of this process, a new set of bounds is obtained, as shown in Fig. 2. This result, which shows a band consis-

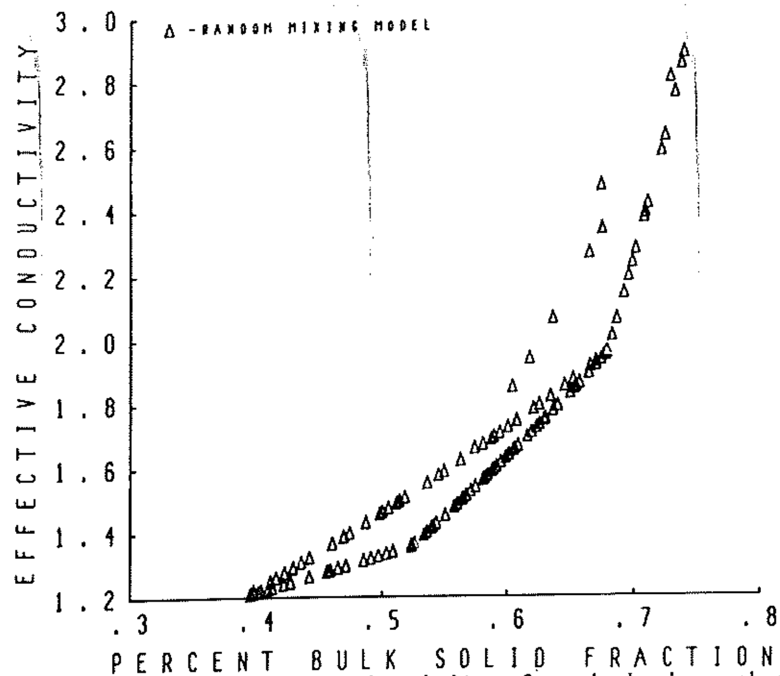

Fig. 2 Effective thermal conductivity of packed microspheres from the random-mixing model.

tent with but narrower than that in Fig. 1, should be a better representation of the actual situation, since it incorporates additional features of randomness in the bed.

Unevacuated Packed Bed of Microspheres

Gaseous conduction must be considered in an unevacuated bed of microspheres. This case, however, can be incorporated easily into the present analytical result. According to the analysis of Ogniewicz and Yovanovich,[5] which also is based on regular-cell packings, the thermal conductivity of a unit cell of microspheres can be expressed in the following form:

$$k_j = K_j[k_s + k_o\ I][(1-\mu^2)P/E]^{1/3} \tag{12}$$

where k_o is the gas conductivity under continuum conditions at STP, and I is an integral function that has been evaluated numerically for various values of the Knudsen number, the ratio of sphere diameter to contact area diameter, and the ratio of solid to gas conductivity. Following the same procedure as previously indicated and defining the dimensionless effective thermal conductivity of an unevacuated bed as

$$k^*_{ue} \equiv k_{ue}/[k_s + k_o\ I][(1-\mu^2)P/E]^{1/3} \tag{13}$$

gives the identical bound as in Eq. (9):

$$\sqrt{K_1 K_2}\ F_1(\delta_v,\alpha,G) \geqslant k^*_{ue} \geqslant \sqrt{K_1 K_2}\ F_2(\delta_v,\alpha,G) \tag{14}$$

A recent study[15] on unevacuated microsphere insulation, however, showed that Kaganer's model[16] for computing the effect of an interstitial gas on the heat transfer in packed beds of granular particles having an internal void structure provides a better correlation with experimental data for hollow, thin-walled microspheres. The bounds in this case remain the same as given in Eq. (14), but the definition of k^*_{ue} will be slightly different.

Two-Dimensional Parallel Fibers

In this model, it is assumed that the needle-shaped fibers are all in a parallel arrangement with each other, re-

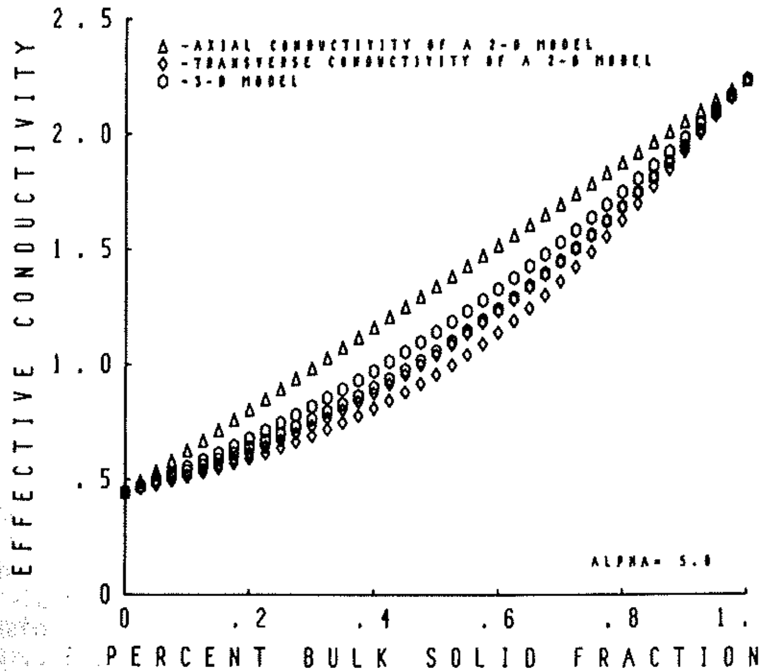

Fig. 3 Effective thermal conductivity of fibrous insulation (α = 5).

sulting in a two-dimensional, anisotropic medium with two effective conductivities in axial and transverse directions. The effective thermal conductivity in the axial direction (e.g., parallel to the fibers) can be obtained readily from simple one-dimensional conduction calculation:

$$k_{e,a}/\sqrt{k_1 k_2} = \delta_v \sqrt{\alpha} + (1-\delta_v)/\sqrt{\alpha} \tag{15}$$

where k_1 and k_2 refer to the conductivities of the fiber and the interstitial material, respectively. For the transverse conductivity, there is a random distribution of circular fibers with a continuum matrix, which, in turn, can be considered as circles of medium composing the matrix. Both have a certain size distribution, so that they will cover the whole fibrous medium. For this two-dimensional cellular model, the bounds on the dimensionless effective transverse conductivity are obtained easily by reducing Beran and Miller's results[9,13] from three- to two-dimensional space:

$$\frac{k_{e,t}}{\sqrt{k_1 k_2}} \leqslant \frac{1 + \delta_v(\alpha-1)}{\sqrt{\alpha}} \left\{ 1 - \frac{[\delta_v(1-\delta_v)(\alpha-1)^2]}{2[1 + \delta_v(\alpha-1)][1 + (\alpha-1)\delta_v + 2(\alpha-1)(1-2\delta_v)G)]} \right\} \tag{16}$$

$$\frac{k_{e,t}}{\sqrt{k_1 k_2}} \geqslant \sqrt{\alpha} \left\{ \alpha - \delta_v(\alpha-1) - \frac{(1-\alpha)^2 \, \delta_v(1-\delta_v)}{2(1-\alpha)\delta_v + 2\alpha + 4(\alpha-1)(2\delta_v-1)G} \right\}^{-1} \tag{17}$$

where G is 1/4 for circle-shaped cells and 1/2 for lamella-shaped cells. For the present case of parallel fibers, G = 1/4. The results of this model are shown in Figs. 3-5 for different values of the ratio of fiber to interstitial-phase conductivities.

Three-Dimensional Dispersed Fibers

Here, the medium is considered to consist of needle-shaped fibers suspended randomly in a continuum matrix. It can be seen that the bounds on the effective conductivity of such a model are given by Eq. (4), since the model is basically a three-dimensional isotropic one. The graphical display of the bounds so obtained also is given in Figs. 3-5. The value of G used here is 1/6 for the needle-shaped cells.

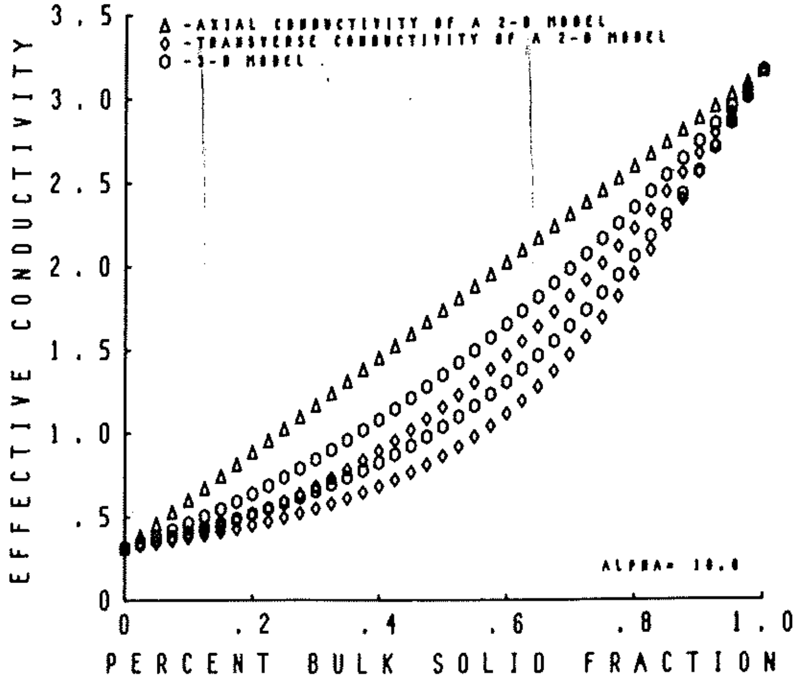

Fig. 4 Effective thermal conductivity of fibrous insulation (α = 10).

Discussion

The most important finding of the present study is the general existence of a well-defined band of effective thermal conductivity values for a particular solid fraction of the microsphere and fibrous insulation. This indicates the necessity of having additional geometric information about the porous medium other than the solid or volume fraction in order to characterize better the effective properties. This information indeed is contained in the general mathematical formulation and, theoretically speaking, can be extracted by carrying out more terms in the perturbation series expansion, thereby introducing the n-point correlation functions.[11,12] From the practical viewpoint, however, these functions are extremely difficult to evaluate when n is greater than 3. It also is not clear whether this complex operation would generate any relevant geometric parameters that are physically meaningful and measurable. Given more geometric or packing information, the bandwidth of the effective value will decrease accordingly. If the complex geometric structure is described completely, the problem becomes, of course, a de-

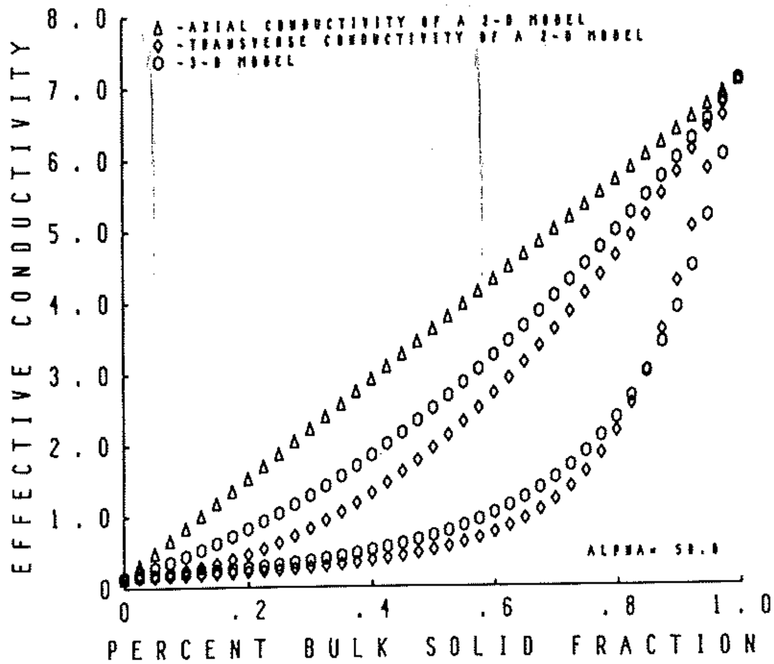

Fig. 5 Effective thermal conductivity of fibrous insulation (α = 50).

terministic one, giving one unique value of the effective thermal conductivity. In general, it is not possible to have such an exact description, but theoretically it is demonstrated clearly in the limit of the close packing (δ_s = 0.74) in the microsphere case, where the band converges to a single point corresponding to the deterministic case of having the closest packing.

The effect of additional geometric information also is exhibited in the results of fibrous insulation. In the two-dimensional model of parallel fibers, by specifying the orientation of fibers in the axial direction and thus introducing more geometric information, the bandwidth is indeed narrower than that of the corresponding three-dimensional model, as shown in Figs. 3-5. The concept of orientation also can be introduced in microspheres by characterizing the orientation of contact points for the packed microsphere. The contact orientation and the coordination number[17] (i.e., the number of contact points for each sphere) appear to be logical geometric parameters, in addition to the solid fraction; however, both are not easily measurable quantities. More studies on the adequate geometric characterization of a porous medium are needed very much.

No comparison of the present results with experimental data is made here because of the lack of data for the effective thermal conductivity at one solid fraction but different packings, as well as at different solid fractions. Existing data normally are obtained for the temperature dependence of the effective thermal conductivity of one particular packing. The present analytical study does demonstrate what experimental data should be taken and how they should be interpreted.

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