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# NONISOTHERMAL CHARACTERIZATION OF THIN FILM OSCILLATING BEARINGS

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***Flow and heat transfer inside a nonisothermal, incompressible, thin-film squeezing bearing are analyzed. The governing equations have been nondimensionalized and reduced to simpler forms based on an order of magnitude analysis. Various analytical solutions for the temperature distribution and Nusselt number under dierent physical constraints are obtained. The influence of the thermal squeezing parameter as well as the motion characteristics of an oscillating bearing are determined on a Nusselt number analytically and numerically, and, similarly, the Nusselt number history is shown for an oscillating thin-film bearing.***

## INTRODUCTION

Thin-film bearings have thicknesses of order of the roughness of the moving surfaces of the bearings. When these thicknesses are filled with fluid, they are said to have fluid-film lubrication. Lubrication usually occurs when the plates of a bearing start to move. Accordingly, the fluid will move by its viscous effect. At the same time the fluid will be compressed due to the load on the bearing. The movement of the fluid creates a self-pumping effect as well as prevents contact between the plates of the bearing. This kind of lubrication is called hydrodynamic lubrication (Sezri [1]). Hydrodynamic lubrication can be found in hydrodynamic journal and thrust bearings. It also can be used in high load capacity applications as discussed by Gross et al. [2].

Self-lubrication in fluid films, like hydrodynamic lubrication, also can be generated by reciprocating or oscillating motions of at least one of the plates of the bearing. These motions in certain intervals will have squeezing effects on the fluid. These will result in pressurizing the fluid due to its viscosity. Accordingly, the fluid will support the load. In the interval when the load is being removed and the plates of the bearing move apart, the fluid will be sucked in and will recover its thickness for the next application. These phenomena are repeated as oscillating motions of the bearing plates continued with no requirement for any external pumping.

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## NOMENCLATURE

| | |
|---|---|
| $B$ | bearing length |
| $c_p$ | specific heat of the fluid |
| $D$ | bearing width |
| $h$ | bearing thickness |
| $H$ | dimensionless bearing thickness |
| $h_c$ | convective heat transfer coefficient |
| $h_o$ | reference bearing thickness |
| $k$ | thermal conductivity of the fluid |
| $p$ | fluid pressure |
| $P_S$ | thermal squeezing parameter |
| $q$ | convective heat transfer |
| $R_L$ | lateral Reynolds number |
| $R_S$ | squeezing Reynolds number |
| $S_1$ | $z$-coordinate of the bearing's lower plate |
| $S_2$ | $z$-coordinate of the bearing's upper plate |
| $\bar{S}_1$ | dimensionless $z$-coordinate of the bearing's lower plate |
| $\bar{S}_2$ | dimensionless $z$-coordinate of the bearing's upper plate |
| $T$ | temperature in fluid |
| $T_1$ | temperature of the bearing's lower plate |
| $T_2$ | temperature of the bearing's upper plate |
| $t$ | time |
| $U$ | dimensionless velocity in the $x$-direction |
| $U_{\bar{S}_1}$ | dimensionless velocity in the $x$-direction at the lower plate |
| $U_{\bar{S}_2}$ | dimensionless velocity in the $x$-direction at the upper plate |
| $u$ | velocity in the $x$-direction |
| $u_{S_1}$ | velocity in the $x$-direction of the lower plate |
| $u_{S_2}$ | velocity in the $x$-direction of the upper plate |
| $V$ | dimensionless velocity in the $y$-direction |
| $V_o$ | dimensionless boundary reference velocity |
| $V_{\bar{S}_1}$ | dimensionless velocity in the $y$-direction at the lower plate |
| $V_{\bar{S}_2}$ | dimensionless velocity in the $y$-direction at the upper plate |
| $v$ | velocity in the $y$-direction |
| $v_{S_1}$ | velocity in the $y$-direction of the lower plate |
| $v_{S_2}$ | velocity in the $y$-direction of the upper plate |
| $W$ | dimensionless velocity in the $z$-direction |
| $\bar{W}$ | average dimensionless velocity in the $z$-direction |
| $W_{\bar{S}_1}$ | dimensionless velocity in the $z$-direction at the lower plate |
| $W_{\bar{S}_2}$ | dimensionless velocity in the $z$-direction at the upper plate |
| $w$ | velocity in the $z$-direction |
| $w_o$ | negative of the dimensional permeable velocity at the bearing's plates |
| $X$ | dimensionless $x$-coordinate |
| $x$ | $x$-coordinate |
| $y$ | $y$-coordinate |
| $Y$ | dimensionless $y$-coordinate |
| $z$ | $z$-coordinate |
| $Z$ | dimensionless $z$-coordinate |
| $\alpha$ | thermal diffusivity |
| $\beta$ | dimensionless squeezing motion amplitude |
| $\varepsilon$ | perturbation parameter |
| $\gamma$ | dimensionless temperature of the upper plate |
| $\lambda$ | dispersion coefficient |
| $\mu$ | dynamic viscosity of the fluid |
| $\theta$ | dimensionless temperature in flow field |
| $\rho$ | density of the fluid |
| $\sigma$ | squeezing number |
| $\tau$ | dimensionless time |
| $\omega$ | reciprocal of a reference time (reference squeezing frequency) |
| $\xi$ | variable transformation for the dimensionless $z$-coordinate |
| $\Pi$ | dimensionless pressure |

The relation between the geometry of the surfaces of a bearing, the relative sliding velocities of these surfaces, the properties of the fluid, and the load on a bearing was first derived by Reynolds [3]. Although he did not include the compressibility effects of the fluid, his derived differential equation was the basic foundation for the fluid-film lubrication theory. There are many studies that have analyzed the flow in hydrodynamic or squeezing lubrication, such as Langlois [4] who solved the momentum equations analytically for hydrodynamic pressure in isothermal squeeze films with fluid density varying according to the pressure. Further Fuller [5] and Cheng [6] studied an average flow model to determine the effects of three-dimensional roughness on partial hydrodynamic lubrication. Later studies considered the influence of heat transfer on the dynamic behavior of a bearing when the fluid viscosity varies with temperature, such as Radakovic and Khonsari [7]. However, these studies did not consider the heat transfer aspects of the bearing.

In this article, the continuity, momentum, and energy equations are set up for a thin-film bearing that has both squeezing and translating motions. Analytical expressions for the velocity field for various conditions are obtained. In addition, the energy equation is solved analytically for the temperature profiles as well as for the Nusselt numbers for the case of steady state one-dimensional flow with bearings having uniform thickness and permeable plates. Also, an analytical solution has been obtained for a limiting case of transient two-dimensional flow with impermeable plates and one plate under oscillatory motion with certain variations of thermal conductivity. Further, the thermal energy equation has been solved numerically for a two-dimensional oscillating bearing with constant thermal conductivity, and a parametric study is performed on this type of bearing.

## FORMULATION OF THE PROBLEM

Consider a thin-film bearing that has a small film thickness $h$ compared with its length $B$ and its width $D$. The lower and upper plates of the bearing are free to move in all directions. The $x$-axis is taken in the direction of the length of the bearing $B$, while $y$ and $z$ axes are taken along the directions of the bearing width $D$ and bearing thickness $h$, respectively, as shown in Figure 1. It is assumed that the fluid is Newtonian and has constant properties. Also, the viscous dissipation is neglected.

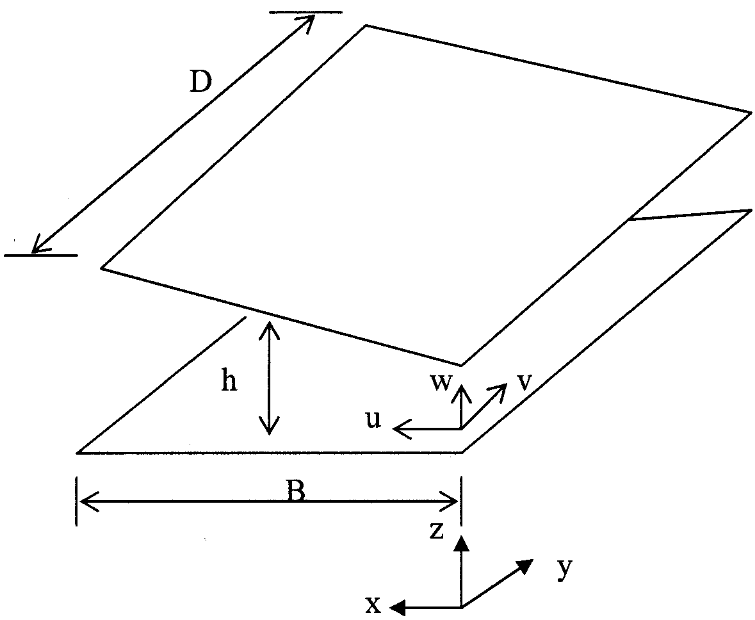


**Figure 1.** Schematic diagram.

## Dimensional Governing Equations

The resulting dimensional continuity, momentum, and energy equations for the bearing are given as Eqs. (1) to (5):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

where $T$, $\rho$, $p$, $\mu$, $c_p$, and $k$ are the fluid temperature, fluid density, pressure, dynamic viscosity of the fluid, specific heat of the fluid, and the thermal conductivity of the fluid, respectively.

A generalized set of boundary conditions for this problem are

$$\begin{aligned} u(x,y,S_1) &= u_{S_1} & u(x,y,S_2) &= u_{S_2} \\ v(x,y,S_1) &= v_{S_1} & v(x,y,S_2) &= v_{S_2} \\ w(x,y,S_1) &= w_{S_1} & w(x,y,S_2) &= w_{S_2} \\ T(x,y,S_1) &= T_1 & T(x,y,S_2) &= T_2 \\ T(0,y,z) &= T_1 & \frac{\partial T(B,y,z)}{\partial x} &= 0 \end{aligned} \tag{6}$$

$S_1$ and $S_2$ in Eq. (6) are the $z$-coordinates of the lower and upper plates, respectively. They could be functions of the other spatial coordinates as well as functions of time. $u_{S_1}$, $u_{S_2}$, $v_{S_1}$, $v_{S_2}$, $w_{S_1}$, $w_{S_2}$, $T_1$, and $T_2$ are either constants, zeros, or functions of spatial coordinates and time. Note that

$$h = S_2 - S_1 \tag{7}$$

## Dimensionless Governing Equations

Equations (1)–(6) are nondimensionalized using the following dimensionless variables:

$$X = \frac{x}{B} \qquad Y = \frac{y}{B} \tag{8(a,b)}$$

$$Z = \frac{z}{h_o} \qquad \tau = \omega t \tag{8(c,d)}$$

$$U = \frac{u}{(\omega B + V_o)} \qquad V = \frac{v}{(\omega B + V_o)} \tag{8(e,f)}$$

$$W = \frac{w}{h_o \omega} \qquad \Pi = \frac{p}{\mu(\omega + V_o/B)\varepsilon^{-2}} \tag{8(g,h)}$$

$$\theta = \frac{T}{T_1} \tag{8(i)}$$

where $\omega$ and $T_1$ are a reference squeezing frequency and the temperature of the lower plate of the bearing, respectively, and $V_o$ is a constant representing a reference dimensional boundary velocity. The variables $X$, $Y$, $Z$, $\tau$, $U$, $V$, $W$, $\Pi$, and $\theta$ are the dimensionless forms of $x$, $y$, $z$, $t$, $u$, $v$, $w$, $p$, and $T$ variables, respectively. The above transformations have been used by Langlois [4] along with the following perturbation parameter:

$$\varepsilon = \frac{h_o}{B} \tag{9}$$

where that $h_o$ is a reference bearing thickness.

Substituting the variables (8) and (9) in Eqs. (1) through (5) and then eliminating the terms that contain $\varepsilon$ raised to the power greater than one except in the energy equation, to show the influence of all defined thermal boundary conditions on thermal characteristics of a thin-film bearing, we obtain the following set of non-dimensionalized governing equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{1}{(1 + V_o/\omega B)} \frac{\partial W}{\partial Z} = 0 \tag{10}$$

$$\frac{\partial \Pi}{\partial X} = \frac{\partial^2 U}{\partial Z^2} - R_S \frac{\partial U}{\partial \tau} - (R_S + R_L)\left[U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y}\right] - R_S W \frac{\partial U}{\partial Z} \tag{11}$$

$$\frac{\partial \Pi}{\partial Y} = \frac{\partial^2 V}{\partial Z^2} - R_S \frac{\partial V}{\partial \tau} - (R_S + R_L)\left[U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y}\right] - R_S W \frac{\partial V}{\partial Z} \tag{12}$$

$$\frac{\partial \Pi}{\partial Z} = 0 \tag{13}$$

$$\frac{\partial^2 \theta}{\partial Z^2} + \varepsilon^2 \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) = P_S \left[\frac{\partial \theta}{\partial \tau} + W \frac{\partial \theta}{\partial Z} + \left(1 + \frac{V_o}{\omega B}\right)\left\{U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y}\right\}\right] \tag{14}$$

where $R_S$, $R_L$, and $P_S$ are squeezing Reynolds number, lateral Reynolds number, and thermal squeezing parameter, respectively. They are given as

$$R_S = \frac{\rho h_o^2 \omega}{\mu} \qquad R_L = \frac{\rho V_o h_o^2}{\mu B} \qquad P_S = \frac{\rho c_p h_o^2 \omega}{k} \tag{15(a,b,c)}$$

It is assumed that both $R_S$ and $R_L$ are of order zero. This is the case for bearings having small film thicknesses and fluids with higher dynamic viscosity and bearing length. The dimensionless value of $h$ is

$$H = \frac{h}{h_o} = \frac{S_2 - S_1}{h_o} = \bar{S}_2 - \bar{S}_1 \tag{16}$$

The corresponding dimensionless boundary conditions will be

$$\begin{aligned}
&U(X, Y, \bar{S}_1) \equiv U_{\bar{S}_1} = \frac{u_{S_1}}{(V_o + \omega B)} \qquad U(X, Y, \bar{S}_2) \equiv U_{\bar{S}_2} = \frac{u_{S_2}}{(V_o + \omega B)}\\
&V(X, Y, \bar{S}_1) \equiv V_{\bar{S}_1} = \frac{v_{S_1}}{(V_o + \omega B)} \qquad V(X, Y, \bar{S}_2) \equiv V_{\bar{S}_2} = \frac{v_{S_2}}{(V_o + \omega B)}\\
&W(X, Y, \bar{S}_1) \equiv W_{\bar{S}_1} = \frac{w_{S_1}}{(h_o\omega)} \qquad W(X, Y, \bar{S}_2) \equiv W_{\bar{S}_2} = \frac{w_{S_2}}{h_o\omega}\\
&\theta(X, Y, \bar{S}_1) = 1.0 \qquad \theta(X, Y, \bar{S}_2) = \gamma\\
&\theta(0, Y, Z) = 1.0 \qquad \frac{\partial\theta(1.0, Y, Z)}{\partial X} = 0
\end{aligned} \tag{17}$$

where $\gamma = T_2/T_1$. Solving Eqs. (11) to (13) for $U$ and $V$, we obtain

$$U = \frac{1}{2}\frac{\partial\Pi}{\partial X}(Z - \bar{S}_1)(Z - \bar{S}_2) + \frac{U_{\bar{S}_1}\bar{S}_2 - U_{\bar{S}_2}\bar{S}_1 + (U_{\bar{S}_2} - U_{\bar{S}_1})Z}{H} \tag{18}$$

$$V = \frac{1}{2}\frac{\partial\Pi}{\partial Y}(Z - \bar{S}_1)(Z - \bar{S}_2) + \frac{V_{\bar{S}_1}\bar{S}_2 - V_{\bar{S}_2}\bar{S}_1 + (V_{\bar{S}_2} - V_{\bar{S}_1})Z}{H} \tag{19}$$

Equations (18) and (19) are the fully developed velocity profiles in the X- and Y-directions, respectively. If these equations are substituted in Eq. (10), the result will be the Reynolds equation given below:

$$\frac{\partial}{\partial X}\left(H^3\frac{\partial\Pi}{\partial X}\right) + \frac{\partial}{\partial Y}\left(H^3\frac{\partial\Pi}{\partial Y}\right) = \sigma\frac{\partial H}{\partial\tau} + 6\left[\frac{\partial}{\partial X}(H(U_{\bar{S}_1} + U_{\bar{S}_2})) + \frac{\partial}{\partial Y}(H(V_{\bar{S}_1} + V_{\bar{S}_2}))\right] \tag{20}$$

where $\sigma$ is the squeezing number and is equal to

$$\sigma = \frac{12}{1 + V_o/\omega B} \tag{21}$$

## Scale Analysis

Consider a two-dimensional infinite thin oscillating bearing where the film thickness $h$ is much greater than the thermal boundary layer thickness $\delta_t$. After the initial squeezing stage, the thermal boundary layer starts to develop during the first stage of the bearing motion from the midsection of the bearing outward to both the left and right sections. For this boundary layer problem, the application of the scale analysis for the continuity Eq. (1) inside the thermal boundary layer reveals that the order of magnitude of the dimensional velocity $u$ is $(\bar{w}B)/h$, where $\bar{w}$ is the wall dimensional squeezing velocity. Note that the order of magnitude of squeezing velocities inside the thermal boundary layer is $(\bar{w}\delta_t)/h$. Utilizing this result along with a scale analysis for the energy Eq. (5), we obtain

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial z^2}\right) \tag{22}$$

This result is obtained by noting that both $w(\partial T/\partial z)$ and $u(\partial T/\partial x)$ are of order $\bar{w}(\Delta T/h)$. Further, the term $\partial^2 T/\partial z^2$, which is of order $\Delta T/\delta_t^2$, is greater than $\partial^2 T/\partial x^2$, which is of order, $\Delta T/B^2$. Accordingly, Eq. (14) reduces to

$$\frac{\partial^2\theta}{\partial Z^2}=P_S\left[\frac{\partial\theta}{\partial\tau}+U\frac{\partial\theta}{\partial X}+W\frac{\partial\theta}{\partial Z}\right] \tag{23}$$

Note that the above analysis shows that the thermal boundary layer thickness is almost independent of the bearing length $B$, and it is valid when $(\delta_t/h)^2 \approx k/\rho c_p\bar{w}h \ll 1$. This is applicable for bearings with large squeezing velocities and operating at relatively large thicknesses as well as for situations where the lubricating fluid has a low thermal diffusivity, that is, for oscillating bearings that have a large thermal squeezing parameter.

## ANALYSIS

### One-Dimensional Steady State with Permeable Boundaries

For the case where the plates of a bearing are fixed, infinite, and permeable, the constant permeable velocity is as follows;

$$W_{\bar{S}_1}=W_{\bar{S}_2}=-a \tag{24}$$

and under steady state conditions Eq. (14) reduces to the following:

$$\frac{\partial^2\theta}{\partial Z^2}+P_S a\frac{\partial\theta}{\partial Z}=0 \tag{25}$$

where $a$ is a constant. Further, the term $P_S a$ is equal to $\frac{h_o w_o}{\alpha}$, where $w_o$ is the negative of the dimensional permeable vertical velocity at the bearing's plates. Equation (25) prescribes a temperature distribution given by

$$\theta(Z)=1-\frac{(\gamma-1)}{e^{-P_S a}-1}\left(1-e^{-P_S aZ}\right) \tag{26}$$

(where $\bar{S}_1$ and $\bar{S}_2$ are taken to be 0 and 1, respectively). The convective heat transfer per unit area of the bearing face is

$$q = h_c(T_2 - T_1) \tag{27a}$$

where $h_c$ is the convective heat transfer coefficient. This convective heat transfer can be calculated based on conduction through the bearing plates as

$$q = -k\left(\left.\frac{\partial T}{\partial z}\right|_{z=S_2} - \left.\frac{\partial T}{\partial z}\right|_{z=S_1}\right) \tag{27b}$$

Accordingly, using Eq. (26) in Eq. (27b), we have that the local Nusselt number for the above case will be

$$\mathrm{Nu} = \frac{h_c h}{k} = P_S a \tag{28}$$

The Nusselt number Nu is positive when more heat is conducted from the fluid to the lower plate than that conducted from the upper plate to the fluid. This is for the values of $T_2$ and initial fluid temperatures that are larger than or equal to $T_1$ and in the absence of any heat source or sink in the fluid. On the other hand, negative Nu values mean that the heat conducted to the fluid from the upper plate is more than the heat transferred from the fluid to the lower plate under the same previous conditions.

## Approximate Solutions for Transient Thin-Film Oscillating Bearing

This solution corresponds to the case where the upper plate of the bearing has a nonzero time varying dimensionless squeezing velocity $W_{\bar{S}_2}$ and the lower plate is fixed. Because of the small thickness of the bearing, the dimensionless velocity of the fluid in the direction of the $z$-axis can be approximated as having a linear trend. Equation (14) is therefore reducible to the following, for $\varepsilon \approx 0$, at dimensionless $X$ values that result in either $U = 0$ or $\partial\theta/\partial X = 0$:

$$\frac{\partial^2\theta}{\partial Z^2} = P_S\left[\frac{\partial\theta}{\partial\tau} + \frac{Z}{H}\frac{dH}{d\tau}\frac{\partial\theta}{\partial Z}\right] \tag{29}$$

where $W = \left(\frac{Z}{H}\right)\frac{dH}{d\tau}$.

The thermal conductivity of the fluid is assumed constant. This is not always true since heat transfer to the fluid is expected to increase at large squeezing velocities as the lubricating fluids contain small particles. These particles at large velocities tend to increase the thermal conductivity due to the dispersion effects [8]. To account for this increase, a simple linear model for the effective thermal conductivity is utilized:

$$k(\tau) = k_o(1 + \lambda|\bar{W}(\tau)|) \tag{30}$$

where $\lambda$ is the dispersion coefficient and $\bar{W}$ is the average of fluid dimensionless squeezing velocities. Accordingly, Eq. (29) changes to

$$(1 + \lambda|\bar{W}(\tau)|)\frac{\partial^2\theta}{\partial Z^2} = P_S\left[\frac{\partial\theta}{\partial\tau} + W\frac{\partial\theta}{\partial Z}\right] \tag{31}$$

The thermal squeezing parameter for this case will be

$$P_S = \frac{\rho c_p h_o^2 \omega}{k_o} \tag{32}$$

Equation (31) reduces, for large squeezing velocities, to

$$\frac{\partial^2\theta}{\partial Z^2} = \frac{P_S}{\lambda}\left[\frac{1}{|\bar{W}(\tau)|}\frac{\partial\theta}{\partial\tau} \pm 2\frac{Z}{H}\frac{\partial\theta}{\partial Z}\right] \tag{33}$$

The positive sign is when the bearing is in its relief stage, while the negative is when it is in the squeezing stage. Introducing a transformation variable $\xi = Z/H(\tau)$ will transform Eq. (33) to

$$\frac{\partial^2\theta}{\partial\xi^2} = 2\frac{P_S}{\lambda}\frac{H^2}{|dH/d\tau|}\frac{\partial\theta}{\partial\tau} \tag{34}$$

The thermal boundary conditions in Eqs. (17) along with the initial condition

$$\theta(Z, 0) = 1 \tag{35}$$

will result in the solution of the dimensionless temperature

$$\theta(\xi,\tau) = 2(\gamma - 1)\left[\sum_{n=1}^{\infty}\frac{(-1)^n}{n\pi}\sin(n\pi\xi)e^{\pm n^2\pi^2\lambda/(2P_S)[1/H - 1/H_o]}\right] + 1 - (1-\gamma)\xi \tag{36}$$

where the positive sign is when the bearing is in its relief stage, while the negative is when it is in the squeezing stage. Accordingly, the Nusselt number can be approximated using the first term of the series

$$\mathrm{Nu}(\tau) = -\frac{4}{H(\tau)}e^{\pm\pi^2\lambda/(2P_S)[1/H(\tau) - 1/H_o]} \tag{37}$$

The previous solution is valid only in the first stage of the bearing motion because $W(\tau)$ and the second term on the right side of Eq. (33) change their sign in the second stage resulting in a new partial differential equation. To solve this new equation, the initial fluid temperature can be replaced by the final temperature from the previous stage, and the result of solving the new equation will have similar terms as in Eqs. (36) and (37) with different coefficients.

### Sinusoidal Squeezing Velocity

Assume the upper plate is moving only in the $z$-direction according to $\bar{S}_2 = H(\tau)$ where

$$H(\tau) = 1 - \beta\cos(\delta\tau) \tag{38}$$

The Nusselt number obtained from Eq. (37) can be written as

$$Nu(\tau) = -4\frac{e^{\left(\frac{\pi^2\beta\lambda}{2PS}\right)\frac{(\cos(\delta\tau)-1)}{(1-\beta)(1-\beta\cos(\delta\tau))}}}{1-\beta\cos(\delta\tau)} \tag{39}$$

This is valid for $0 < \delta\tau < \pi$, since it will result in $W > 0$. The negative values of Nu indicate that heat conduction at the upper plate is larger than that at the lower plate.

The dimensionless velocity components $U$ and $W$ can be obtained from Eqs. (18), (20), and (10), knowing that $\sigma$ is equal to 12 for pure oscillatory motion of the upper plate. They are

$$U(X,\xi,\tau) = 6\delta\beta\frac{\sin(\delta\tau)}{1-\beta\cos(\delta\tau)}X\xi(\xi-1) \tag{40}$$

$$W(\xi,\tau) = -6\delta\beta\sin(\delta\tau)\xi^2\left(\frac{\xi}{3}-\frac{1}{2}\right) \tag{41}$$

These equations are valid for $\partial\Pi/\partial X|_{X=0} = 0$. Accordingly, the previous solution is valid at $X = 0$ and $X = 1$ where the axial temperature gradient is assumed to be zero.

## TWO-DIMENSIONAL OSCILLATING BEARING

In this section, Eq. (14) is transformed to $X$, $\xi$ and $\tau$ domains. The resulting equation is

$$\frac{\partial^2\theta}{\partial\xi^2} + \varepsilon^2H^2\frac{\partial^2\theta}{\partial X^2} = P_S\left[H^2\frac{\partial\theta}{\partial\tau} + \left(W - \xi\frac{dH}{d\tau}\right)H\frac{\partial\theta}{\partial\xi} + UH^2\frac{\partial\theta}{\partial X}\right] \tag{42}$$

This equation along with Eqs. (40) and (41) were solved numerically for a two-dimensional oscillating bearing with a fixed lower plate and the upper one being moved according to Eq. (38). The transformed thermal boundary conditions and the initial condition implemented are

$$\theta(X,0,\tau) = 1.0 \tag{43}$$

$$\theta(X,1,\tau) = \gamma \tag{44}$$

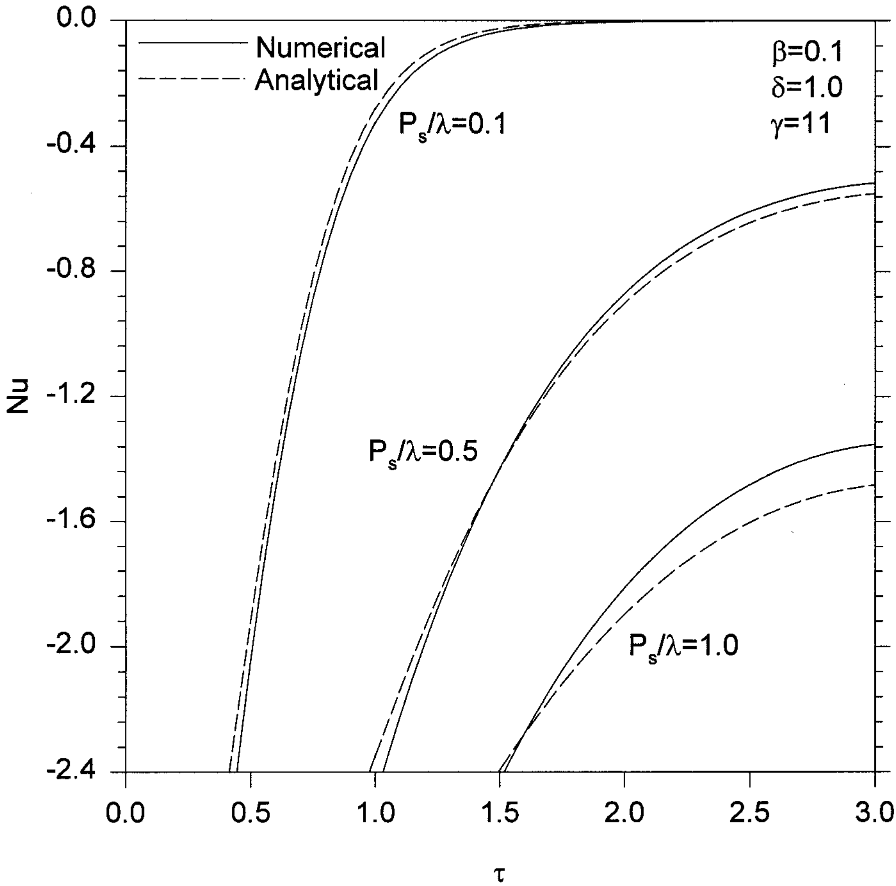


**Figure 2.** Effects of $P_S$ and dispersion on Nusselt number for an oscillating bearing.

$$\theta(0, \xi, \tau) = 1.0 \tag{45}$$

$$\frac{\partial \theta(1.0, \xi, \tau)}{\partial X} = 0 \tag{46}$$

$$\theta(X, \xi, 0) = 1.0 \tag{47}$$

The numerical results are shown in Figures 3–7, where the average Nusselt number is the average of

$$\mathrm{Nu}(X, \tau) = \frac{-1}{(\gamma - 1)H(\tau)} \left[ \frac{\partial \theta}{\partial \xi}\bigg|_{\xi=1} - \frac{\partial \theta}{\partial \xi}\bigg|_{\xi=0} \right] \tag{48}$$

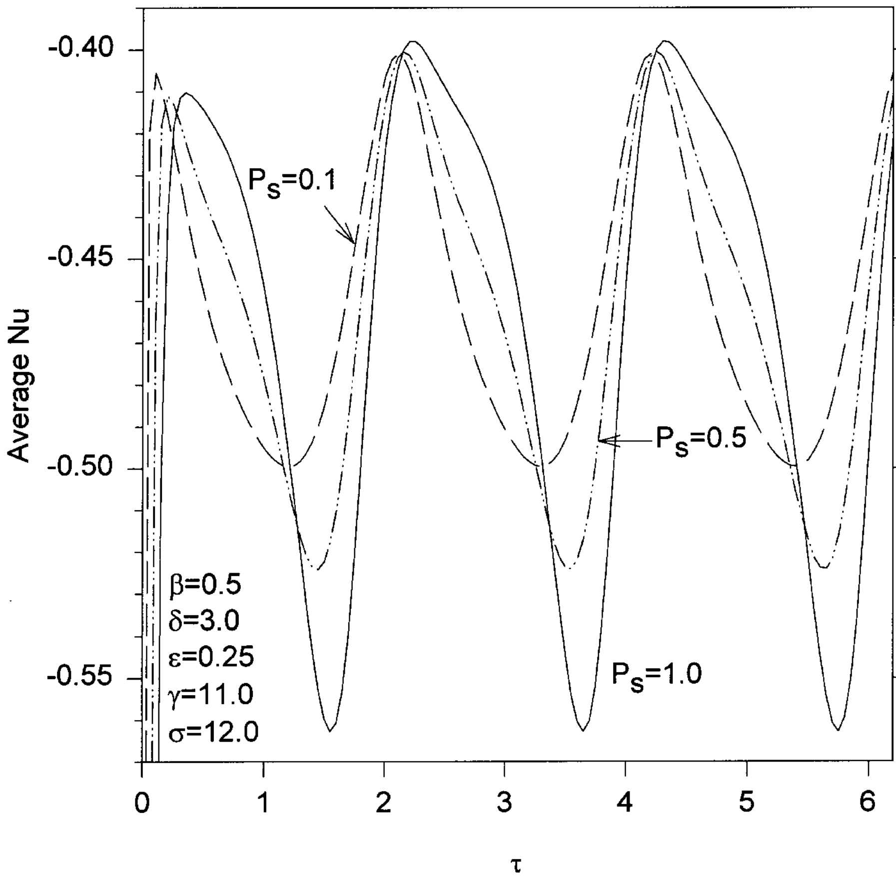


**Figure 3.** Effects of $P_S$ and dimensionless time on Nusselt number for an oscillatory bearing (low $P_S$).

## NUMERICAL ANALYSIS

Equation (42) was solved using the implicit finite difference scheme. Center differencing in space was used for discretizing the dimensionless temperature differential terms, except for the terms $\partial\theta/\partial X$ and $\partial^2\theta/\partial X^2$ at $X = 1$, where backward differencing was used, and forward differencing was used to approximate the time differential term. Further, the value of $\delta$ in Eq. (38) is chosen to be 3.0 in the solution to the two-dimensional oscillating bearing in order to reduce the computational time. This value makes thermal squeezing parameters, see Eq. (15*c*) shown in Figures 3–7, equal to one third of their values when dimensional variables in Eqs. 8(*a*–*i*) are normalized by the actual squeezing frequency.

The one-dimensional case represented by Eq. (34) was solved using the previous implicit finite difference scheme, and the obtained results were found to be in excellent agreement with the derived approximate analytical solution in the sinusoidal squeezing velocity section. Accordingly, the reduced energy equation was solved for various characteristics of an oscillatory bearing with constant thermal conductivity in order to better understand the bearing's thermal response under different conditions.

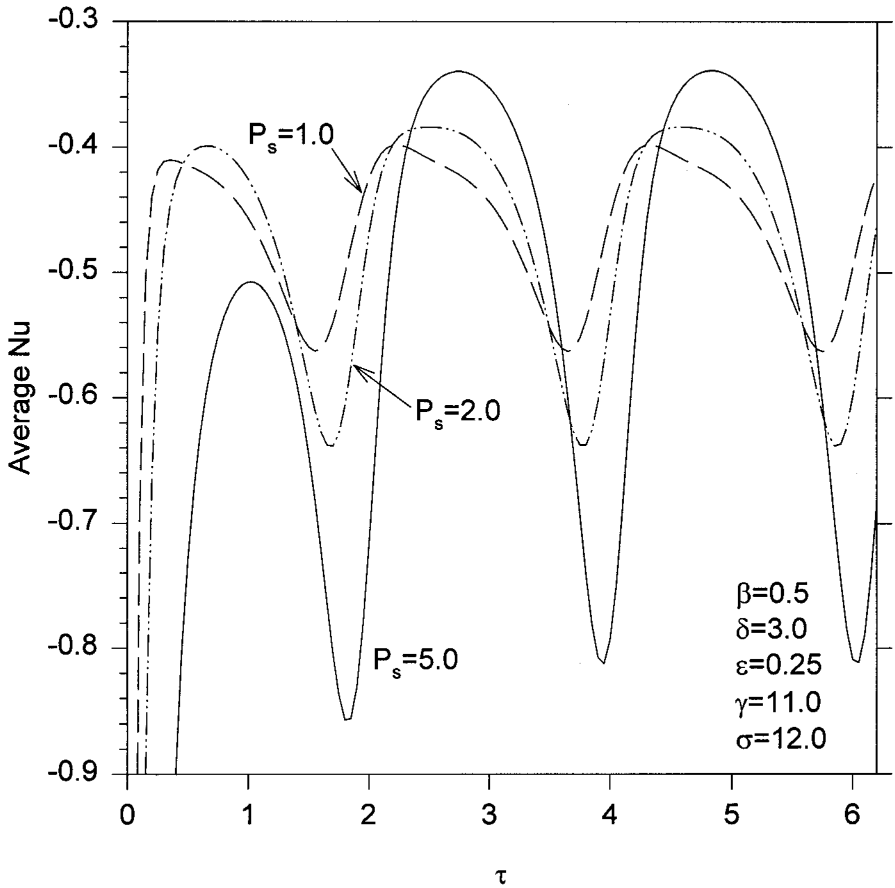


**Figure 4.** Effects of $P_S$ and dimensionless time on average Nusselt number for an oscillatory bearing at large $P_S$.

## DISCUSSION OF RESULTS

Equation (28) suggests that the Nusselt number Nu increases as both the dimensionless permeable velocity $a$ and the thermal squeezing parameter $P_S$ increase. The value of thermal squeezing parameter $P_S$ is increased only if the bearing fluid thermal diffusivity $\alpha$ is reduced and if the dimensional squeezing velocities are increased as in the case for high duty operations.

Figure 2 represents the behavior of the proposed oscillating bearing described by Eq. (38) under conditions where the fluid encounters dispersion effects due to the presence of small-scale particles such that its thermal conductivity is modeled by Eq. (30). The results of solving Eq. (34) for Nusselt numbers are shown in Figure 2. It is noticed that both the approximate analytical and numerical solutions are in excellent agreement. Further, it is noticed that the Nusselt number is always negative and decreasing asymptotically to zero. This can be noticed from Eq. (37).

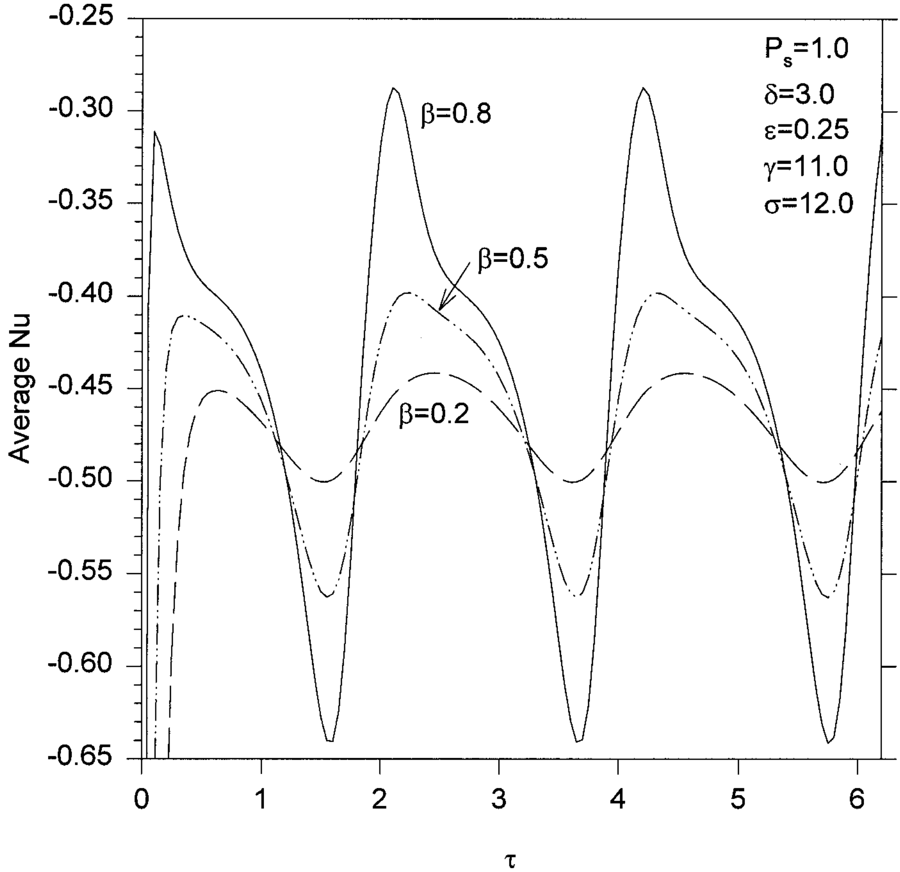


**Figure 5.** Effects of β and dimensionless time on average Nusselt number for an oscillatory bearing ($P_S$ = 1.0).

Figures 3 and 4 represent the effects of thermal squeezing parameter $P_S$ on the average Nusselt number for an oscillating bearing with dimensionless frequency $\delta = 3.0$ with constant thermal conductivity. It is noticed that the frequency of the Nusselt number is similar to the frequency of the upper plate motion. The increase in $P_S$ results in enhancing the convection inside the bearing as predicted from Eq. (42), and this causes increases in the absolute values of average Nu as shown in Figures 3 and 4. Further, it is noticed from Eq. (42) that the coefficient of the second term on the right is relatively smaller than the coefficient of the third term on the same side. Also, the coefficient of the second term has similar values with different signs at points having similar distances from the lower and upper plates. This indicates that the average Nu values in an oscillating bearing are mainly influenced by axial convection and conduction mechanisms.

The absolute values of average Nu are maximum in squeezing stages as shown in Figures 3 and 4. This can be interpreted by the fact that induced velocities in squeezing stages are directed outward from the bearing end that has the minimum

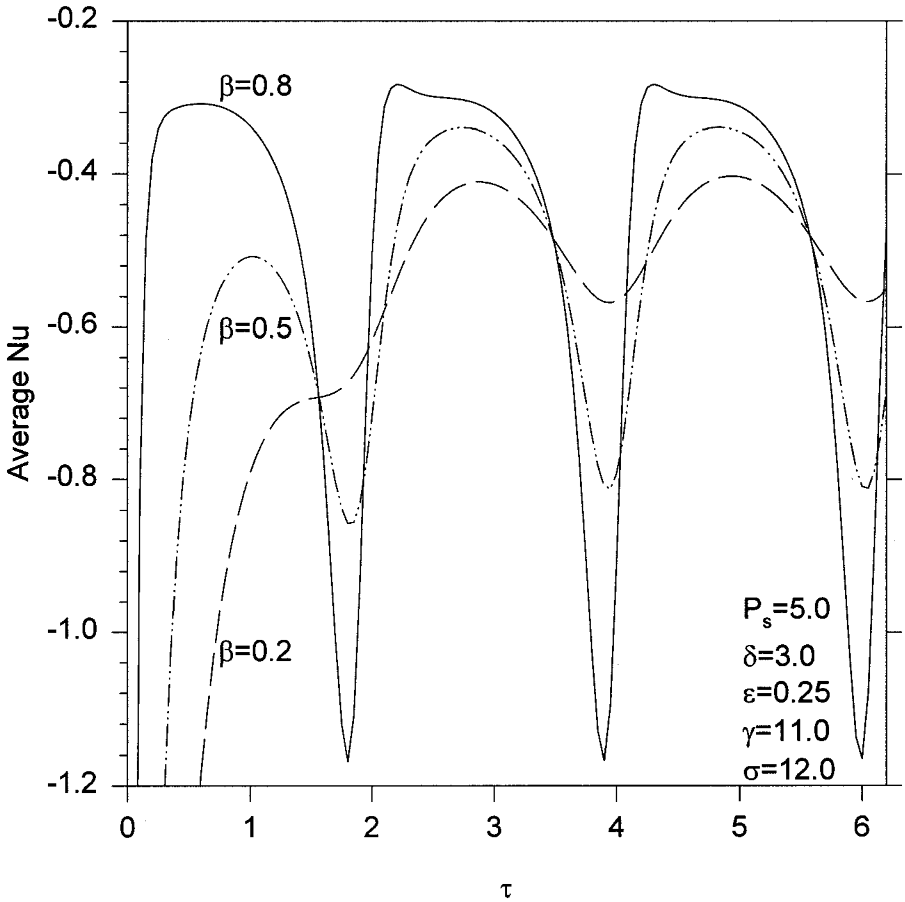


**Figure 6.** Effects of β and dimensionless time on average Nusselt number for an oscillatory bearing ($P_S$ = 5.0).

temperature in the field. Thus, the heat transfer from the upper plate is expected to increase due to enhancements in the convective heat transfer. On the other hand, absolute values of average Nu are minimum when induced velocities are directed toward that end since they result in increasing the average fluid temperatures. Accordingly, the heat transfer from the upper plate is reduced. In addition, the peaks of average Nu values are observed to occur at periods where the thickness has almost its average value for larger values of $P_S$ because $U$ and $W$ are maximum at these points; thus convection is maximized as can be seen from Eqs. (40) and (41). This is not the case for lower values of $P_S$ since the conduction is dominant in these applications.

Figures 5 and 6 show the effects of the dimensionless motion amplitude β of the upper plate on the average Nu for two different values of $P_S$ with a constant thermal conductivity. It is observed that increases in the values of β result in increases in the absolute values of the average Nu in squeezing stages due to increased induced

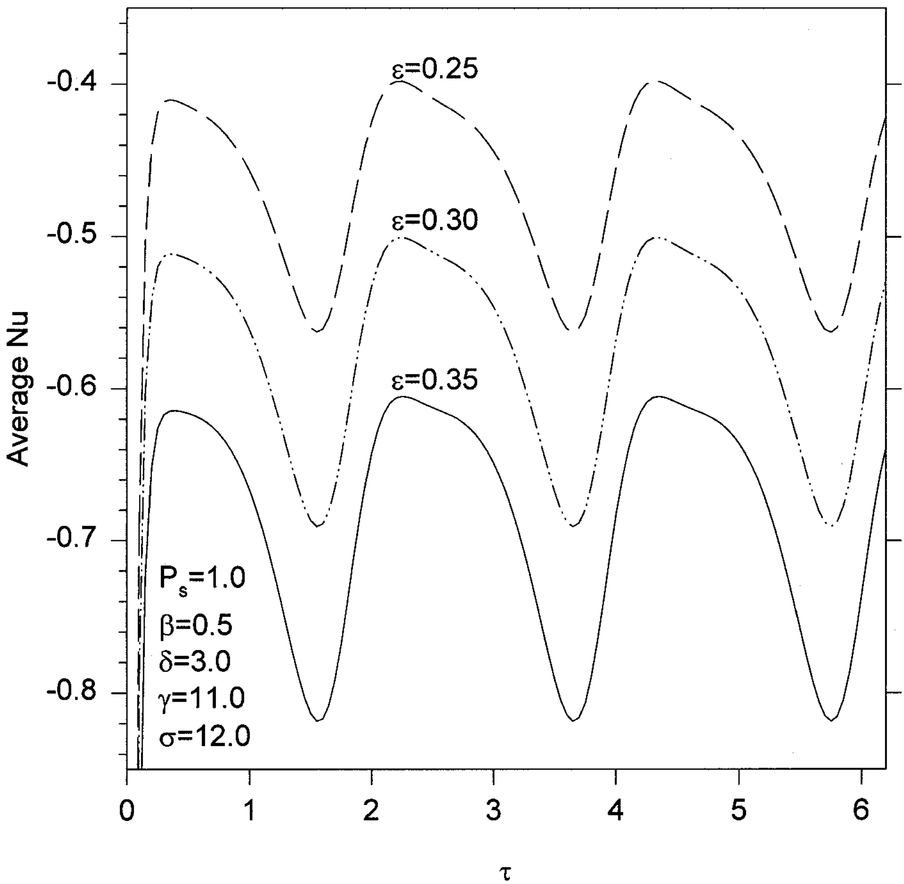


**Figure 7.** Effects of ε and dimensionless time on average Nusselt number for an oscillatory bearing.

velocities as β increases. However, the increased induced velocities in relief stages due to increases in β values result in increases in average fluid temperatures, which cause average absolute Nu values to decrease. Further, it is noticed that the trend of the absolute average Nu values becomes flatter during relief stages as β increases, as shown from Figure 6. This is because convective terms shown in Eq. (42) increase as β increases. However, these convective terms become smaller during squeezing stages. Accordingly and due to the presence of large heat transfer enhancements during squeezing stages, the trend of the average Nu values becomes sharper during squeezing stages as β increases as shown in Figure 6.

Figure 7 shows the effect of the perturbation parameter ε on the average Nu values. It is shown that as ε increases, the absolute values of the average Nu increase. As ε increases, axial conduction increases, resulting in increases in average Nu values. It is worth noting that as ε increases, two-dimensional effects on velocity profiles increase and solutions for the complete momentum equations are needed. Finally, enhancements of heat transfer inside an oscillating bearing can be achieved by the following: increasing the amplitude of the motion of the oscillating plate, considering an oscillating bearing with large thermal squeezing parameter, and pumping fluid with minimum temperature to the bearing during relief stages.

## CONCLUSIONS

The flow and heat transfer effects of boundary squeezing of the plates of an incompressible thin-film bearing have been considered in this work. Although flow inside bearings has been studied in the past, the heat transfer characteristics of oscillatory bearings have not been studied. The proper energy equation and its limiting cases were obtained. The reduced energy equation has been solved for one-dimensional steady state heat transfer in an infinite bearing with both permeable plates and uniform thickness. Also, an approximate solution was obtained for the two-dimensional transient heat transfer in oscillating bearings having variable thermal conductivity. This approximate solution was compared with the numerical solution of the corresponding thermal energy equation. The results were found to be in excellent agreement. Further, the energy equation along with the velocity field for an oscillatory squeezing bearing with fluid having constant thermal conductivity is solved. It was found that the oscillating dynamic behavior of the bearing also results in an oscillatory thermal behavior of the bearing. Further, it was shown that the amplitude of the average Nusselt number is increased by an increase in the thermal squeezing parameter, the motion amplitude of the bearing, and the perturbation parameter. This study can lead the way for further investigations on the effect of nonisothermal thin films.

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