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NONISOTHERMAL CHARACTERIZATION OF THIN-FILM OSCILLATING BEARINGS IN THE PRESENCE OF ULTRAFINE PARTICLES

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The effects of viscous dissipation and thermal dispersion due to the presence of ultrafine particles in the lubricating fluid are studied on flow and heat transfer inside a nonisothermal and incompressible bearing during relief and squeezing stages. The governing equations are nondimensionlized and reduced to simpler forms based on an order-of-magnitude analysis. Analytical solution for the energy equation for a special case is obtained. Further, the influence of the thermal squeezing parameter, Eckert number, thermal dispersion coefficient, the motion characteristics of an oscillating bearing, and the perturbation parameter are determined. It is shown that the average heat transfer parameter decreases by an increase in the thermal squeezing parameter and dispersion coefficient, while it increases as both the Eckert number and amplitude motion parameter are increased.

INTRODUCTION

Self-lubrication in bearings can be generated by oscillating motions of bearing plates. These motions will pressurize the fluid due to its viscosity in squeezing intervals causing the fluid to support the load. In relaxed intervals, when the plates of the bearing move apart, the fluid will be sucked in and will recover its thickness for the next application (see Sezri [1] and Gross et al. [2]). These phenomena are repeated as oscillating motions of the bearing plates continue, with no requirement for any external pumping.

Many studies have analyzed the flow in hydrodynamic or squeezing lubrication, such as Langlois [3], who solved the momentum equations analytically for the hydrodynamic pressure in isothermal squeeze films with fluid density varying as a function of the pressure. Further, Fuller [4] and Cheng [5] studied an average flow model to determine effects of three-dimensional roughness on partial hydrodynamic lubrication.

Later studies considered the influence of heat transfer on the dynamic behavior of a bearing when the fluid viscosity varies with temperature. Radakovic and

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NOMENCLATURE

B	bearing length	x	x coordinate
c_p	specific heat of the fluid	X	dimensionless x coordinate
h	bearing thickness	y	y coordinate
h_o	reference bearing thickness	Y	dimensionless y coordinate
H	dimensionless bearing thickness	α	thermal diffusivity
k	thermal conductivity of the fluid	β	dimensionless squeezing motion amplitude
p	fluid pressure		
P_s	thermal squeezing parameter	ε	perturbation parameter
q	net local heat flux transferred to the lubricating fluid	θ	dimensionless temperature in flow field
		Θ	local dimensionless heat parameter
R_s	squeezing Reynolds number	λ	thermal dispersion coefficient
t	time	μ	dynamic viscosity of the fluid
T	temperature in fluid	Φ	dimensionless pressure
T_1	temperature of the bearing's lower plate	ρ	density of the fluid
T_2	temperature of the bearing's upper plate	τ	dimensionless time
u	velocity in the x direction	ξ	variable transformation for the dimensionless z-coordinate
U	dimensionless velocity in the x direction		
v	velocity in the y direction	ω	reciprocal of a reference time (reference squeezing frequency)
V	dimensionless velocity in the y direction		

Khonsari [6] considered the effects of heat transfer and viscous dissipation on the dynamics of piston rings. Wang et al. [7] performed a thermodynamic analysis on journal bearings lubricated with fluids having couple stresses. However, these studies did not consider the heat transfer aspects of the bearing. Recently, Khaled and Vafai [8] considered heat transfer in incompressible squeeze thin films, but they did not include effects of viscous dissipation or the possibility of presence of nanoparticles in the lubricating fluid.

Heat transfer is found to be enhanced if nanoparticles are suspended in the lubricating fluid. This is seen in the works of Xuan and Li [9] and Eastman et al. [10]. This is because nanoparticles tend to increase the exposed heat transfer surface area and the heat capacity of the fluid. Further, the presence of nanoparticles in the lubricating fluid increases the mixing within the fluid, which causes an additional increase in the fluid's thermal conductivity due to thermal dispersion effects, as discussed by Xuan and Li [9].

In this work, the continuity, momentum, and energy equations for a thin-film bearing having a pure squeezing motion are transformed into a dimensionless form while taking into account the presence of both thermal dispersion and viscous dissipation effects. Analytical expressions for the velocity field are obtained. Further, heat transfer is determined analytically for a limiting case of transient two-dimensional oscillating bearing in the presence of viscous dissipation. Finally, the transformed thermal energy equation is solved numerically and a parametric study for various characteristics of the bearing is performed.

FORMULATION OF THE PROBLEM

A two-dimensional thin-film bearing having a small film thickness h compared to its length B is shown in Figure 1. The x and y axes are taken in the direction of the

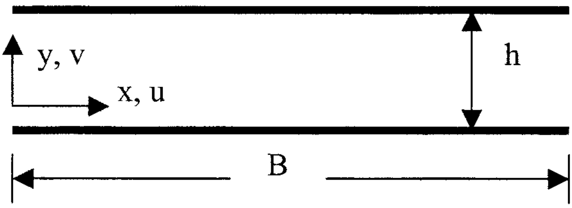

Figure 1. Schematic diagram.

length of the bearing B and along the bearing thickness h, respectively, as shown in Figure 1. The lower plate of the bearing is fixed and the upper plate of the bearing is moving according to the following relation:

$$h = h_o[1 - \beta\cos(\delta\omega t)] \tag{1}$$

where h_o, ω, β, and δ are a reference bearing thickness, a reference frequency, the dimensionless amplitude of the motion, and a constant, respectively. It is assumed that the fluid is Newtonian and has constant properties except for its thermal conductivity. Also, the viscous dissipation inside the lubricating fluid is considered.

Dimensional Governing Equations

The corresponding dimensional continuity, momentum, and energy equations for the bearing are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{4}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left\{\left(\frac{\partial u}{\partial y}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]\right\} \tag{5}$$

where T, ρ, p, μ, c_p, and k are the fluid temperature, fluid density, pressure, dynamic viscosity of the fluid, specific heat of the fluid, and the thermal conductivity of the fluid, respectively.

The dimensional boundary conditions are

$$
\begin{aligned}
&u(x,0,t) = 0 && u(x,h,t) = 0 \\
&\frac{\partial p(0,y,t)}{\partial x} = 0 && \\
&v(x,0,t) = 0 && v(x,h,t) = h_o \omega \beta \delta \sin(\delta \omega t) \\
&T(x,0,t) = T_1 && T(x,h,t) = T_2 \\
&T(0,y,t) = T_1 && \frac{\partial T(B,y,t)}{\partial x} = 0 \\
&T(x,y,0) = T_1 &&
\end{aligned}
\tag{6}
$$

where T_1 and T_2 are constants.

Dimensionless Governing Equations

The following dimensionless variables [3] are used to dimensionlize Eqs. (1)–(6):

$$X = \frac{x}{B} \qquad Y = \frac{y}{h_o} \qquad 7(a,b)$$

$$\tau = \omega t \qquad U = \frac{u}{\omega B} \qquad 7(c,d)$$

$$V = \frac{v}{h_o \omega} \qquad \Pi = \frac{p}{\mu \omega \varepsilon^{-2}} \qquad 7(e,f)$$

$$\theta = \frac{T - T_1}{T_2 - T_1} \qquad 7(g)$$

where T_1 and θ are the temperature of the lower plate of the bearing and the dimensionless temperature, respectively. The variables X, Y, τ, U, V, and Π are the dimensionless forms of x, y, t, u, v, and p variables, respectively. We define the perturbation parameter as

$$\varepsilon = \frac{h_o}{B} \tag{8}$$

Nondimensionlizing Eqs. (2)–(4) and using Eq. (7) and applying the perturbation parameter given by Eq. (8) results in

$$U = \frac{1}{2} \frac{\partial \Pi}{\partial X} Y(Y - H) \tag{9}$$

$$\frac{\partial}{\partial X}\left(H^3 \frac{\partial \Pi}{\partial X}\right) = 12 \frac{\partial H}{\partial \tau} \tag{10}$$

where

$$H \equiv \frac{h}{h_o} = 1 - \beta\cos(\delta\tau) \tag{11}$$

Equation (10) represents the reduced Reynolds equation for an oscillating bearing (see Sezri [11]). Further, the above solutions are valid for low values of both the squeezing Reynolds number and the perturbation parameter. The squeezing Reynolds number is

$$R_s = \frac{\rho h_o^2 \omega}{\mu} \tag{12}$$

The solution to Eqs. (9) and (10) results in the following approximate velocity profiles:

$$U(X, Y, \tau) = 6\delta\beta \frac{\sin(\delta\tau)}{[1 - \beta\cos(\delta\tau)]^3} XY(Y - H) \tag{13}$$

$$V(Y, \tau) = -6\delta\beta \frac{\sin(\delta\tau)}{[1 - \beta\cos(\delta\tau)]^3} Y^2 \left(\frac{Y}{3} - \frac{H}{2}\right) \tag{14}$$

The thermal conductivity of the fluid is considered variable, because the existence of ultrafine particles in lubricating fluids in certain applications is expected to enhance the heat transfer in these fluids at large squeezing velocities. These enhancements can be shown with metallic nanoparticles in the works of Xuan and Li [9] and Eastman et al. [10]. Further, the work of Adams et al. [12] shows that enhancements to the heat transfer also occur in the presence of dissolved air molecules in fluids. These ultrafine particles, at large velocities, tend to increase the thermal conductivity due to thermal dispersion effects. To account for this increase, a linear model between the effective thermal conductivity and the fluid speed is utilized [13]:

$$k(X, Y, \tau) = k_o[1 + \lambda\sqrt{U^2(X, Y, \tau) + \varepsilon^2 V^2(X, Y, \tau)}] = k_o\phi(X, Y, \tau) \tag{15}$$

where λ is the thermal dispersion coefficient and is linearly proportional to ωB. k_o is a reference thermal conductivity of the lubricating fluid that contains ultrafine particles. This reference thermal conductivity is usually greater than the thermal conductivity of the pure fluid [8]. Equation (5) is reduced to the following when dimensionless variables [Eqs. (7) and (8)] are used:

$$P_s\left(\frac{\partial\theta}{\partial\tau} + U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y}\right) = \varepsilon^2\frac{\partial}{\partial X}\left(\phi\frac{\partial\theta}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\phi\frac{\partial\theta}{\partial Y}\right) + E_s\left\{\left(\frac{\partial U}{\partial Y}\right)^2 + 2\varepsilon^2\left[\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right]\right\} \tag{16}$$

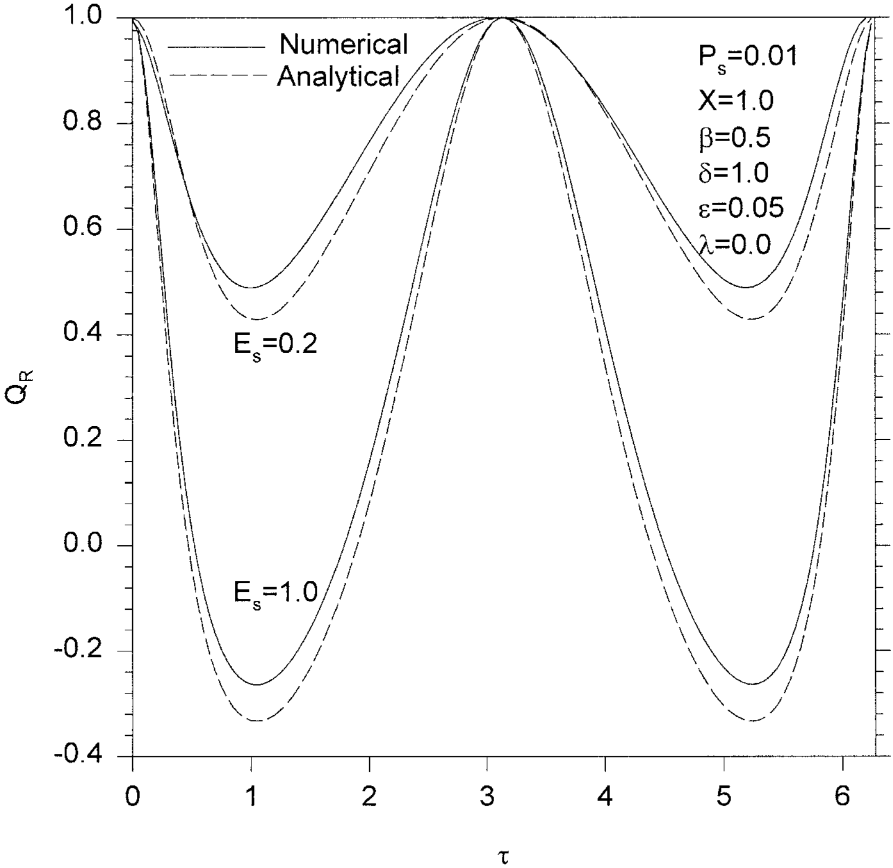

Figure 2. Effects of E_S on heat ratio for an oscillating bearing for negligible P_S.

where P_s and E_s are the thermal squeezing parameter and the Eckert number, respectively,

$$P_s = \frac{\rho c_p h_o^2 \omega}{k_o} \qquad E_s = \frac{\mu \omega^2 B^2}{k_o(T_2 - T_1)} \tag{17}$$

Note that E_s is proportional to the square of the reference frequency ω and P_s is proportional to the reference frequency ω. The corresponding dimensionless thermal boundary and initial conditions are

$$\begin{aligned} &\theta(X,0,\tau) = 0 \qquad \theta(X,1,\tau) = 1 \\ &\theta(0,Y,\tau) = 0 \qquad \frac{\partial \theta(1,Y,\tau)}{\partial X} = 0 \\ &\theta(X,Y,0) = 0 \end{aligned} \tag{18}$$

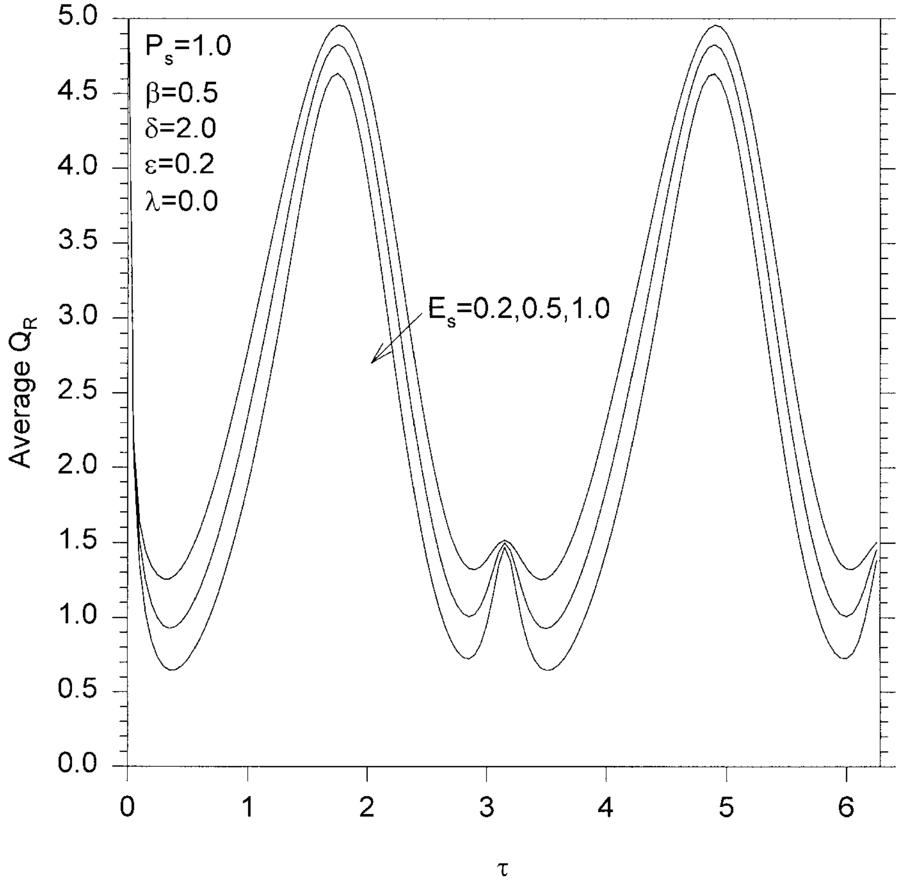

Figure 3. Effects of E_S on average heat ratio for an oscillating bearing at $P_S = 1.0$.

Introducing the variable $\xi = Y/H(\tau)$ transforms Eqs. (16) to

$$P_s\left[H^2\frac{\partial\theta}{\partial\tau} + UH^2\frac{\partial\theta}{\partial X} + \left(V - \xi\frac{dH}{d\tau}\right)H\frac{\partial\theta}{\partial\xi}\right] = \varepsilon^2 H^2\frac{\partial}{\partial X}\left(\phi\frac{\partial\theta}{\partial X}\right)$$

$$+\frac{\partial}{\partial\xi}\left(\phi\frac{\partial\theta}{\partial\xi}\right) + E_s\left\{\left(\frac{\partial U}{\partial\xi}\right)^2 + 2\varepsilon^2\left[H^2\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial\xi}\right)^2\right]\right\} \tag{19}$$

Further, velocity profiles given by Eqs. (13) and (14) in the X, ξ, and τ domain are

$$U(X, \xi, \tau) = 6\delta\beta\frac{\sin(\delta\tau)}{1 - \beta\cos(\delta\tau)}X\xi(\xi - 1) \tag{20}$$

$$V(\xi, \tau) = -6\delta\beta\sin(\delta\tau)\xi^2\left(\frac{\xi}{3} - \frac{1}{2}\right) \tag{21}$$

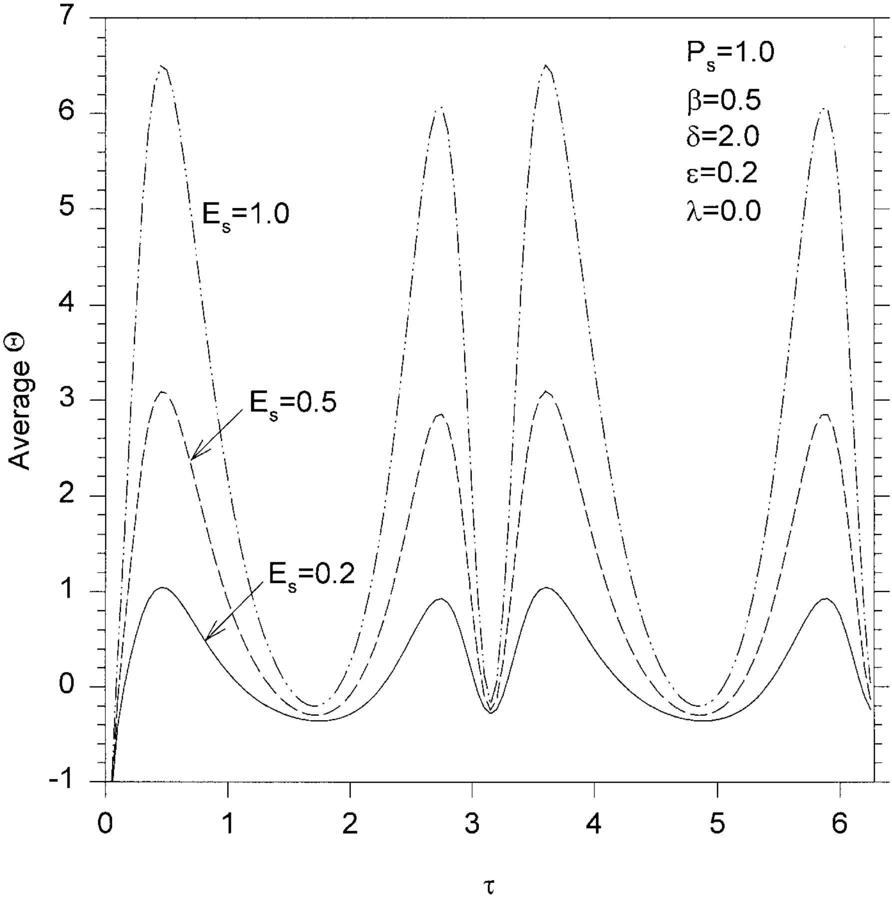

Figure 4. Effects of E_S on average dimensionless heat parameter for an oscillating bearing at $P_S = 1.0$.

The ratio Q_R is defined as the ratio of the heat transfer at the upper plate to that at the lower plate. It can be calculated from the following:

$$Q_R(X, \tau) = (1 + \lambda\varepsilon|\delta\beta \sin(\delta\tau)|) \left. \frac{\partial\theta(X, \xi, \tau)}{\partial\xi} \right|_{\xi=1} \Big/ \left. \frac{\partial\theta(X, \xi, \tau)}{\partial\xi} \right|_{\xi=0} \tag{22}$$

As expected, viscous dissipation results in an increase in the average fluid temperature, and this ensures that heat transfer to the lower plate is always negative. Accordingly, negative Q_R values indicate that both upper and lower plates are gaining heat due to large viscous dissipation. Positive values of Q_R indicates that the upper plate is losing more heat than the lower plate gains if Q_R is greater than one, and vice versa when Q_R is less than one.

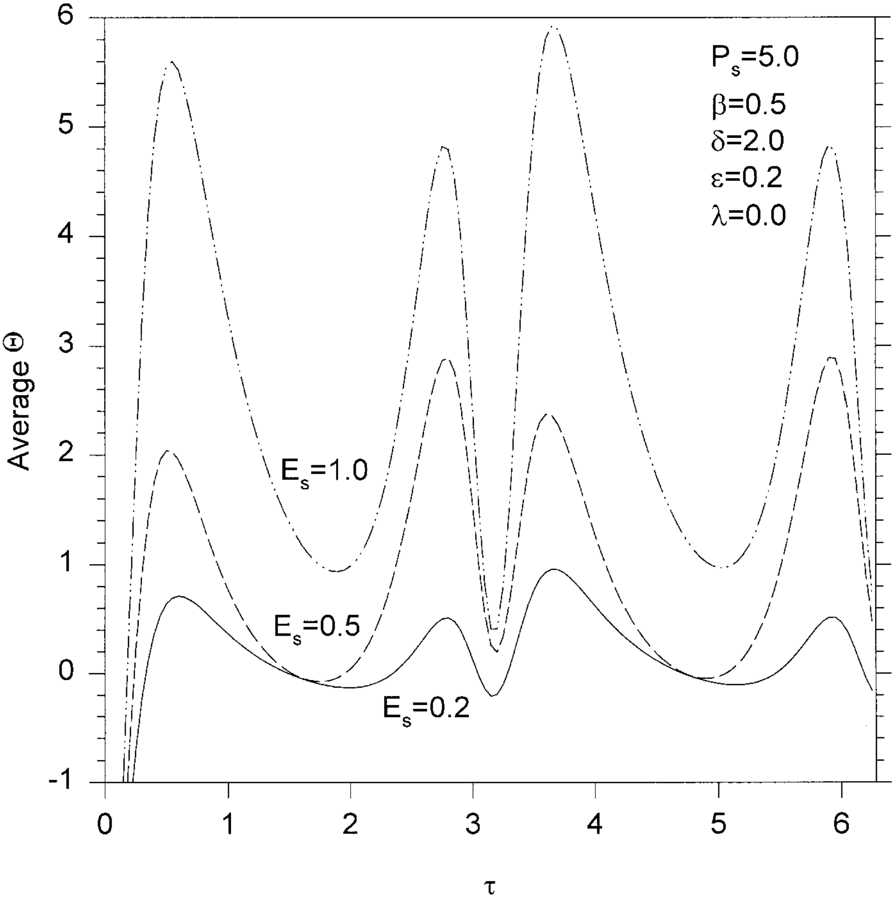

Figure 5. Effects of E_S on average dimensionless heat parameter for an oscillating bearing at $P_S = 5.0$.

Another important factor in thermal characteristics of lubricating bearings is the local dimensionless heat parameter Θ, which represents the net local dimensionless heat transfer to the lubricating fluid. The local dimensionless heat parameter Θ can be related to the Q_R ratio by the following relations:

$$\Theta(X,\tau) \equiv \frac{q(X,\tau)h_o}{k_o(T_2 - T_1)} = -\frac{1}{k_o}\left[\left(k\frac{\partial\theta(X,Y,\tau)}{\partial Y}\right)\bigg|_{Y=H} - \left(k\frac{\partial\theta(X,Y,\tau)}{\partial Y}\right)\bigg|_{Y=0}\right] \quad 23(a)$$

$$\Theta(X,\tau) = -\frac{1}{H}\frac{\partial\theta(X,\xi,\tau)}{\partial\xi}\bigg|_{\xi=0}(Q_R - 1) \quad 23(b)$$

where $q(X,\tau)$ is the net dimensional local heat flux that is transferred to the fluid. When Θ is negative, more heat will be transferred to the lubricating fluid. However, positive values of Θ indicate that the heat added to the lower plate is more than that

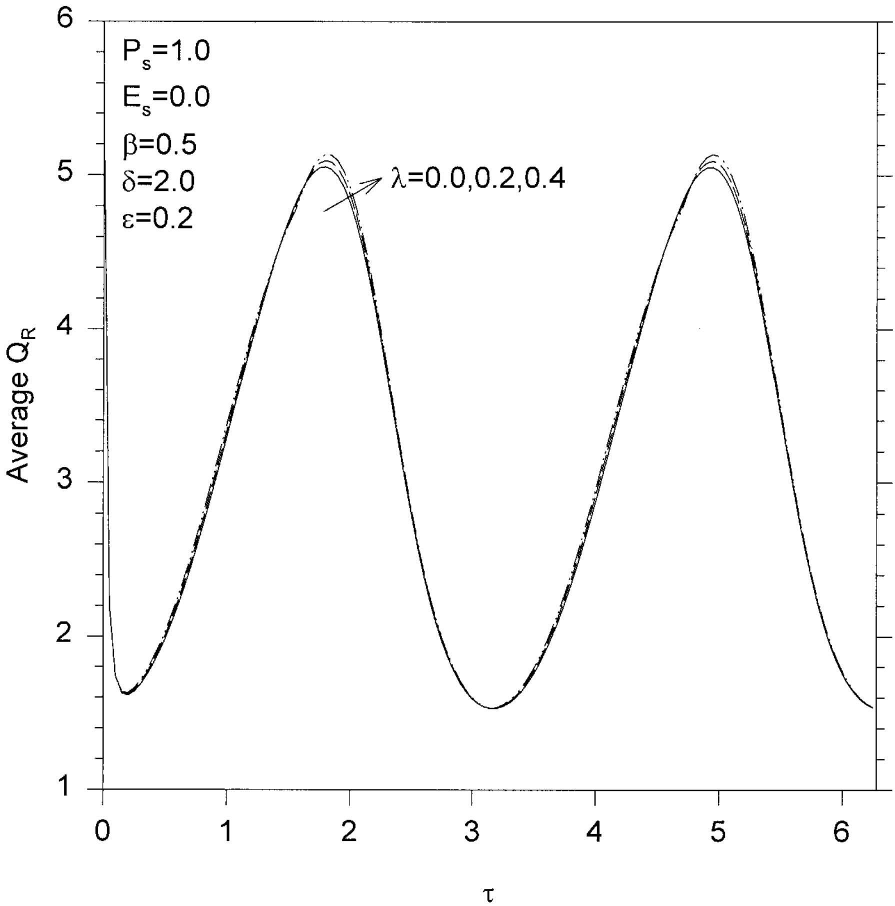

Figure 6. Effects of λ on average heat ratio for an oscillating bearing at $P_S = 1.0$.

lost from the upper plate as long as the values of Q_R are above zero. Note that the values of

$$\frac{1}{H}\frac{\partial\theta}{\partial\xi}\bigg|_{\xi=0}$$

are always positive in absence of heat absorption in the fluid and when T_2 is greater than T_1. When Q_R is decreased below zero, both upper and lower plates will be gaining heat, as in cases where large viscous dissipation is present.

Approximate Solution for Oscillating Bearing with Viscous Dissipation

For small values of the perturbation ε, Eq. (19) can be approximated by the following at $X = 1$:

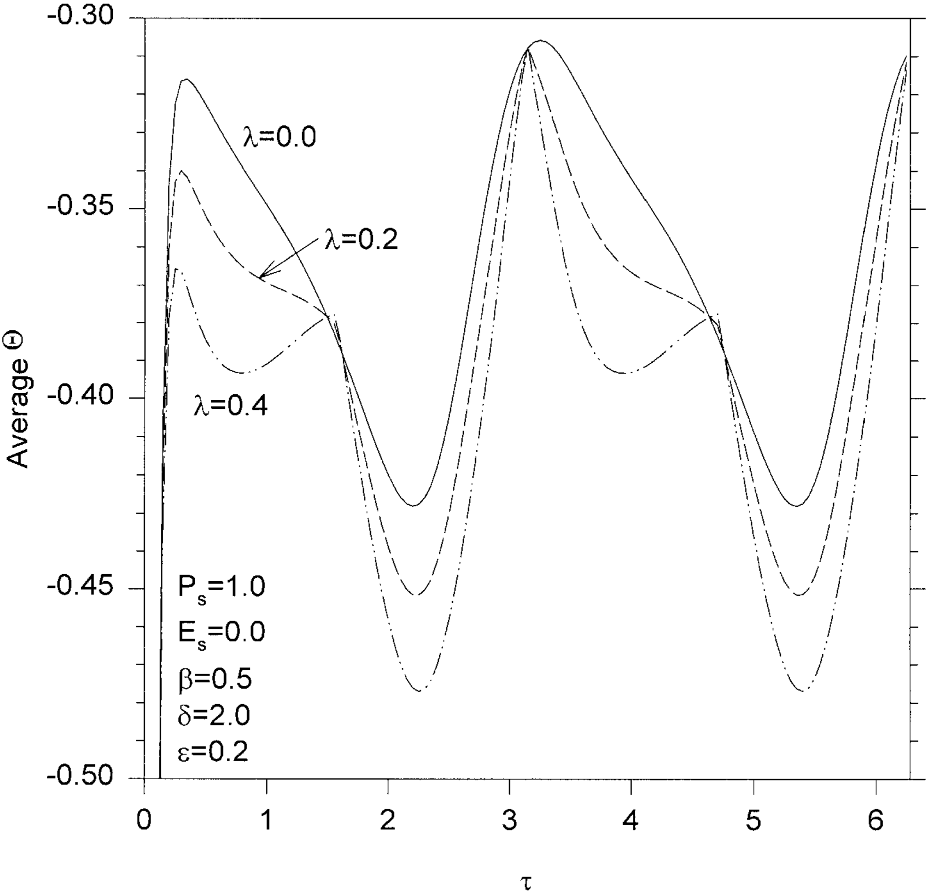

Figure 7. Effects of λ on average dimensionless heat parameter for an oscillating bearing at $P_S = 1.0$.

$$P_s H^2 \frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial\xi}\left(\phi\frac{\partial\theta}{\partial\xi}\right) + E_s\left(\frac{\partial U}{\partial\xi}\right)^2 \tag{24}$$

Q_R values for the above equation for both small thermal squeezing parameter and small thermal dispersion coefficient are

$$Q_R = \frac{[1-\beta\cos(\delta\tau)]^2 - 6E_s\beta^2\delta^2\sin^2(\delta\tau)}{[1-\beta\cos(\delta\tau)]^2 + 6E_s\beta^2\delta^2\sin^2(\delta\tau)} \tag{25}$$

Note that as E_s gets larger, Q_R becomes negative.

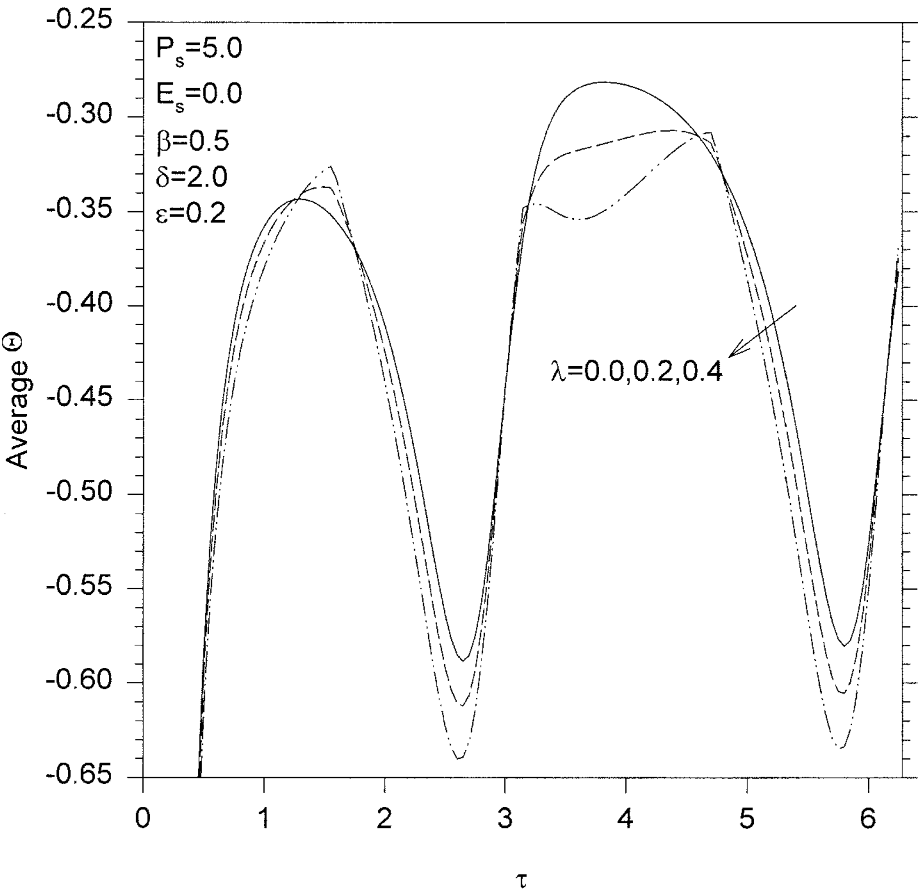

Figure 8. Effects of λ on average dimensionless heat parameter for an oscillating bearing at $P_S = 5.0$.

NUMERICAL ANALYSIS

Equation (19) was solved using an implicit finite-difference scheme. Center differencing in space was used for discretizing the dimensionless temperature differential terms, except for the terms $\partial\theta/\partial X$ and $\partial^2\theta/\partial X^2$ at $X = 1$, where backward differencing was used, and forward differencing was used to approximate the time differential term. The value of δ in Eq. (1) is chosen to be 2.0 in Figures 3–10. Other values of δ resulted in similar physical behavior as shown in Figures 3–10. This value makes the thermal squeezing parameters shown in Figures 3–10 equal to half their values when dimensional variables in Eqs. (7) are normalized by the actual squeezing frequency. Based on extensive numerical experimentation, the values of 0.0125, 0.04, and 0.05 are chosen for ΔX, $\Delta\xi$, and $\Delta\tau$, respectively. These values result in grid and time independence. It is worth noting that both average Q_R and average Θ represented in Figures 3–10 are averages of the local Q_R and Θ values, respectively, after $X = 0.0125$. Moreover, the initial conditions are set slightly greater than zero to avoid divisions by zero in Q_R.

The results obtained for the heat ratio were found to be in good agreement with the results obtained from Eq. (25). Accordingly, the reduced energy equation was solved for various values of the thermal squeezing parameter, Eckert number, thermal dispersion coefficient, amplitude of the upper plate's motion, and perturbation parameter in order to better understand the bearing's thermal response under different conditions.

DISCUSSION OF RESULTS

Figure 2 represents the behavior of a thin-film oscillating bearing with negligible thermal squeezing parameter ($P_s = 0.01$) in the presence of viscous dissipation. It is noticed that the numerical values of the local Q_R are in good agreement with the analytical results of Eq. (25). Further, it is noticed that the values of Q_R are always less than one because the generated heat due to viscous dissipation tends to reduce the heat transfer at the upper plate. Negative values of Q_R are achieved for large values of Eckert number E_s, thus both upper and lower plates are gaining heat.

Figure 3 represents the effects of viscous dissipation on the average heat ratio Q_R for an oscillating bearing with dimensionless frequency $\delta = 2.0$ with constant thermal conductivity. It is noticed that the frequency of the average Q_R is similar to the frequency of the upper plate motion. Further, it is noticed from this figure that the average Q_R decreases as E_s increases. Effects of viscous dissipation on the average dimensionless heat parameter Θ are shown in Figures 4 and 5 for two different values of thermal squeezing parameter. The increase in P_s results in enhancing the convection inside the bearing at constant E_s number, as predicted from Eq. (19). Accordingly, this results in a decrease in the average values of Θ as shown in Figures 4 and 5. In addition, the following can be noticed from Figures 3–5:

The maximum average Q_R and the minimum average Θ are found to occur in the early squeezing stages.

The minimum average Q_R and maximum average Θ are found to occur at times that makes the induced horizontal velocities reach almost their maximum and minimum values due to increases in viscous dissipation.

Figure 6 shows the effects of thermal dispersion coefficient λ on the average heat ratio Q_R in the absence of viscous dissipation. It is noticed that the average Q_R increases as λ increases and these increases in the average Q_R are significant early during the squeezing stage. The effects of thermal dispersion coefficient λ on the average Θ are seen in Figures 7 and 8 for two different values of thermal squeezing parameters. The following are observed from Figures 7 and 8:

The average Θ decreases as λ increases. This is expected due to enhancements in the fluid thermal conductivity.

The significant enhancements in the average value of Θ occur during the squeezing stage. This is because induced velocities are directed outward from the bearing's end, which has the minimum temperature. During the relief stage, the induced velocities tend to increase the average fluid temperatures and this causes enhancements during the relief stage to be lower than those during the squeezing stage.

The performance of the bearing is improved during the relief stage due to enhancements in the thermal conductivity.

Figure 9 represents the influence of the motion amplitude β on the average dimensionless heat parameter Θ. Induced velocities increase by an order of β as β increases. Meanwhile, viscous dissipation increases by an order of β^2 as β increases. Accordingly, as seen in Figure 9, the values of the average Θ are expected to increase as β increases.

Figure 10 illustrates the effects of the perturbation parameter on the average dimensionless heat parameter Θ. As ε increases, fluid axial conduction increases, resulting in heat transfer enhancements to the fluid. Accordingly, the average value of Θ decreases as shown in Figure 10. It is worth noting that as ε increases,

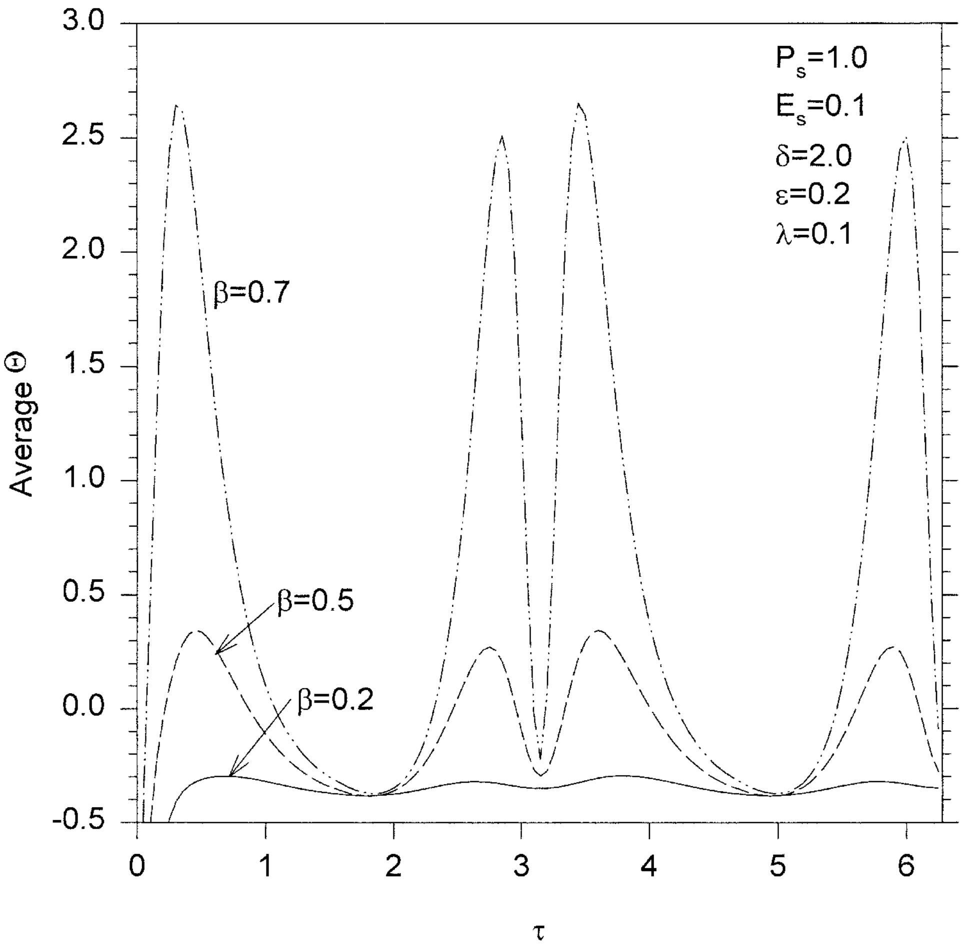

Figure 9. Effects of β on average dimensionless heat parameter for an oscillating bearing.

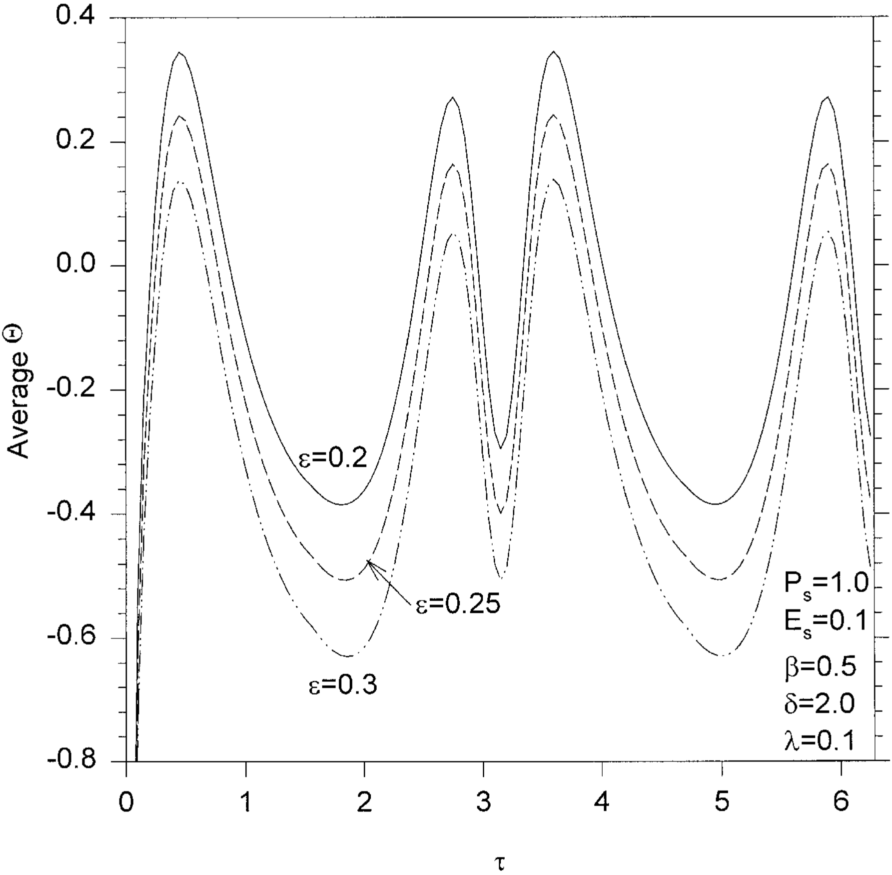

Figure 10. Effects of ε on average dimensionless heat parameter for an oscillating bearing.

two-dimensional effects on velocity profiles increase. Finally, enhancements of heat transfer inside the oscillating bearing can be achieved by introducing nanoparticles in the lubricating fluid, selecting a the lubricating fluid that will result in a minimum Eckert number, considering an oscillating bearing with large thermal squeezing parameter, and increasing the motion amplitude if Eckert number is negligible.

CONCLUSIONS

The flow and heat transfer effects during the squeezing and relief stages for an incompressible thin-film bearing have been considered in the presence of both thermal dispersion and viscous dissipation. Although flow inside bearings has been studied in the past, the heat transfer characteristics of oscillatory bearings have received less attention, especially with presence of viscous dissipation or thermal dispersion. In the present work, the proper energy equation was dimensionlized and an approximate solution was obtained for a special case of two-dimensional transient heat transfer in oscillating bearings in the presence of viscous dissipation. This approximate solution was compared with the numerical solution of the corresponding thermal energy equation. The results were found to be in good agreement. Further, the reduced transformed energy equation along with the velocity field for an oscillatory squeezing bearing was solved. It was found that the oscillating dynamic behavior of the bearing also results in oscillatory thermal behavior of the bearing. It was found that the average dimensionless heat parameter decreases with an increase in the thermal squeezing parameter, thermal dispersion coefficient, and the perturbation parameter, while it increases as both the Eckert number and amplitude motion parameter are increased.

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