On the Linear Encroachment in Two-Immiscible Fluid Systems in a Porous Medium

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A careful review to the previous study of Srinivasan and Vafai on the linear encroachment in two-immiscible fluid systems in a porous medium reveals some typos in their analytical solution. In the present study, an accurate analytical solution, which accounts for boundary and inertia effects, is obtained to predict the movement of the interfacial front and corrections to previous results are provided wherever necessary. Despite the similarity in the general behavior of the present accurate solution and the previous one, the existence of an accurate analytical solution is essential for future numerical and experimental studies.
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Introduction

Linear encroachment of two-immiscible fluid systems is an important problem in many engineering applications such as die filling and injection molding. Muskat [1] considered a one-dimensional Darcy's flow model to study the linear encroachment of two fluids in a narrow channel. An analytical solution for linear encroachment in two-immiscible fluid systems in a porous material was presented in Srinivasan and Vafai [2] where boundary and inertia effects are accounted for. It is found that using this analytical solution is required in the vicinity of solid boundaries and/or faster moving flows where Muskat's model fails to predict the necessary time for the encroaching fluid to completely displace the second fluid. In addition, Srinivasan and Vafai [2] show that implementing their analytical solution is essential for cases of low mobility ratio as well as higher values of permeability.

A schematic diagram of the flow configuration and the coordinate system is shown in Fig. 1. Governing equations and boundary conditions for the problem under consideration are provided in Srinivasan and Vafai [2]. Unfortunately a small typo in the exact solution caused a defection in some of the results. The purpose of this note is to correct these typos and adjust the dependent results accordingly.

Results

The same procedures used in [2] are used here to correct the analytical solution. As mentioned earlier, all governing equations and boundary conditions given in [2] are correct. The first typo is found in Eq. (21), which should read as follows:

$$u_D=\frac{K_1}{\mu_1}\frac{P_{in}-P_e}{(1-\epsilon)X_0^M-L} \tag{1}$$

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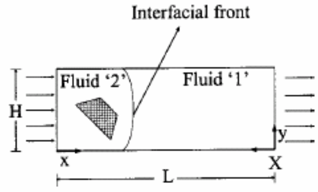

Fig. 1 Schematic diagram of the problem

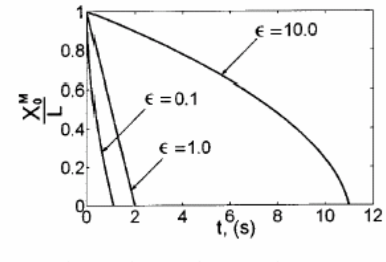

Fig. 2 Prediction of the location of interface using Darcy's model for different mobility ratios "ϵ" with δ=0.45, ΔP=2.25 $\times 10^7$, μ_1=0.01002, K=1$\times 10^{-10}$, and L=1 m

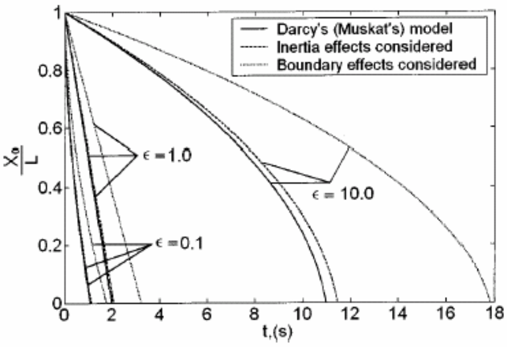

Fig. 3 Progress of the interfacial front at $y=\sqrt{K/\delta}$ (η=1), δ=0.45, ΔP=2.25$\times 10^7$, μ_1=0.01002, K=1$\times 10^{-10}$, and L=1 m using different models

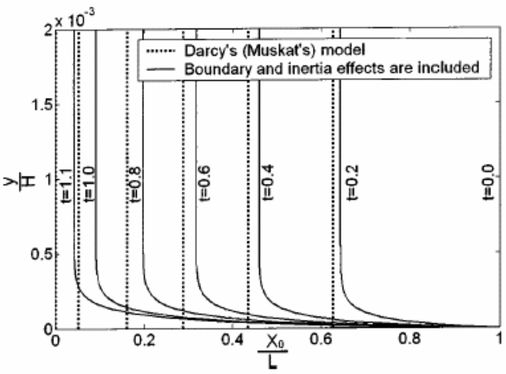

Fig. 4 Progress of the interfacial front, ϵ=0.1, δ=0.45, ΔP =2.25$\times 10^7$, μ_1=0.01002, K=1$\times 10^{-10}$, and L=1 m

Table 1 Summary of the corrections

Present	Srinivasan and Vafai [2]
$\frac{X_0^M}{L}=\frac{1-\sqrt{\epsilon^2+(1-\epsilon)\left\{\frac{2K_1}{\mu_1\delta}\frac{P_{in}-P_e}{L^2}t\right\}}}{(1-\epsilon)}$	$\frac{X_a^M}{L}=\frac{-\epsilon+\sqrt{1-(1-\epsilon)\left\{\frac{2K_1}{\mu_1\delta}\frac{P_{in}-P_e}{L^2}t\right\}}}{(1-\epsilon)}$
$u_D=\frac{K_1}{\mu_1}\frac{P_{in}-P_e}{(1-\epsilon)X_0^M-L}$	$u_D=\frac{K_1}{\mu_1}\frac{P_{in}-P_e}{(1-\epsilon)X_0^M+\epsilon L}$

There are two typos in Eq. (23), which should read as follows:

$$\left(\frac{X_0^M}{L}-1\right)\left(\frac{X_0^M}{L}(1-\epsilon)-(1+\epsilon)\right)-\frac{2K_1}{\mu_1\delta}\frac{(P_{in}-P_e)}{L^2}t=0. \quad (2)$$

As a result, the obtained analytical solution in [2] (Eq. (24)) should be replaced by

$$\frac{X_0^M}{L}=\frac{2-\sqrt{4+4(1-\epsilon)\left\{\frac{2K_1}{\mu_1\delta}\frac{P_{in}-P_e}{L^2}t-(1+\epsilon)\right\}}}{2(1-\epsilon)}. \quad (3)$$

The singular perturbation used to find the inertial and boundary effects in [2] are found to be accurate. This makes it easy to incorporate the expression of the velocity in the boundary region as well as the expression of the velocity in the core region to the analytical solution in Eq. (3). Using these expressions along with the analytical solution for the Darcian free-surface front position, the analytical solution inside the boundary layer region and the analytical solution in the core region, respectively, are

$$(1-\epsilon)\left(\frac{X_0^M}{L}\right)^2-2\left(\frac{X_0^M}{L}\right)-\left\{\frac{2K_1}{\mu_1\delta}\frac{(P_{in}-P_e)}{L^2}(u_b^0+\delta u_b^1+\delta^2u_b^2+\ \dots)t-(1+\epsilon)\right\}=0 \quad (4)$$

$$(1-\epsilon)\left(\frac{X_0^M}{L}\right)^2-2\left(\frac{X_0^M}{L}\right)-\left\{\frac{2K_1}{\mu_1\delta}\frac{(P_{in}-P_e)}{L^2}(1-\beta\delta+2\beta^2\delta^2-5\beta^3\delta^3)t-(1+\epsilon)\right\}=0. \quad (5)$$

The above two equations (4), (5) are to replace Eqs. (25) and (26) in [2]. Figures 2–4 are to replace Figs. 2, 4, and 5 in [2], respectively. Other Figs. in [2] are not corrected here for the sake of brevity. In fact, Figs. 6, 7 in [2] are similar to Fig. 5 while results in Fig. 3 can be deduced from Eq. (1). Finally, a summary of the main findings is shown in Table 1.

Conclusions

Analytical solution for the problem of linear encroachment in two-immiscible fluid systems in a porous material is obtained. The main characteristics of the previous results in [2] are unaffected. However, it is believed that the existence of an accurate analytical solution is essential for future numerical and experimental studies.

Nomenclature

- H = width of the channel (m)
- K = permeability of the porous structure (m^2)
- L = horizontal extent of the channel (m)
- P = pressure (Nm^{-2})
- P_e = pressure at exit (Nm^{-2})
- P_{in} = pressure at inlet (Nm^{-2})
- t = time (s)
- u_b = dimensionless velocity field in the boundary layer region (ms^{-1})
- u_D = Darcian convective velocity (ms^{-1})
- x = coordinate along the horizontal length of the channel from left to right as shown in Fig. 1 (m)
- X = coordinate along the horizontal length of the channel from right to left as shown in Fig. 1 (m)
- X_0 = location of the interface using the generalized model (m)
- X_0^M = location of the interface using the Muskat's model (m)
- y = coordinate along the vertical length of the channel (m)
- β = parameter defined as the product of the Reynolds number and the empirical function F
- δ = porosity of the porous medium
- η = dimensionless vertical coordinate
- μ = fluid viscosity ($kg\,m^{-1}\,s^{-1}$)
- ϵ = mobility ratio

References

[1] Muskat, M., 1937, *The Flow of Homogeneous Fluids Through Porous Media*, 1st Ed., Edwards, Ann Arbor, MI.

[2] Srinivasan, V., and Vafai, K., 1994, "Analysis of Linear Encroachment in Two-Immiscible Fluid Systems in a Porous Medium," ASME J. Fluids Eng., **116**, pp. 135–139.