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# Using measurements in urban areas to estimate turbulent velocities for modeling dispersion

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# Abstract

This study extends a study [Princevac, M., Venkatram, A., 2007. Estimating micrometeorological inputs for modeling dispersion in urban areas during stable conditions. Atmospheric Environment, doi:10.1016/j.atmosenv.2007.02.029.] in which mean winds and temperatures measured at one or two levels on towers located in urban areas were fitted to Monin–Obukhov similarity equations to obtain estimates of micrometeorological variables required in modeling dispersion in the stable boundary layer. This study shows that such methods are also useful in unstable conditions: measurements of the mean wind speed and the standard deviation of temperature fluctuations,  $\sigma_{\rm T}$ , at one level on a tower yield estimates of surface heat flux, surface friction velocity, and standard deviations of turbulent velocities that are within a factor of two of values observed at two urban sites over 80% of the time.

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#### 1. Introduction

This paper extends the examination (Princevac and Venkatram, 2007) of the performance of three methods to estimate the surface friction velocity and the Monin–Obukhov (MO) length in stable conditions when the meteorological measurements are made at urban locations that do not meet the criteria for application of MO theory. Estimates from these methods were compared with measurements made at two urban sites: the Wilmington site located in the middle of an urban area, and the

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Vertical Transport and Mixing Experiment (VTMX) site located on a sloping, smooth area in Salt Lake City (SLC). The first method used the mean wind at a single height (Single U or SU), the second used the wind speed at a single level and the temperature difference between two levels (U delta T or UDT), and the third method used two levels of wind speed and temperature (delta U delta T or DUDT). The performance of the SU and UDT methods in estimating  $u_*$  were comparable. The SU method yielded better estimates of the MO length than the UDT method does. The DUDT method performed poorly in estimating both  $u_*$  and L. The major conclusions of this study were that (1) measurements of mean winds and temperatures at one or two levels at an urban location can

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provide adequate estimates of micrometeorological variables required in modeling dispersion in the stable boundary layer, and (2) methods based on using differences in temperatures and velocities between two levels can provide unreliable estimates of these variables because these differences can be overwhelmed by inevitable uncertainties in the measurement of mean variables.

Like the previous study, this study is also motivated by the need for meteorological inputs for dispersion models such as AERMOD (Cimorelli et al., 2005). The question we attempt to answer here is: Do MO similarity methods that apply to flat terrain provide useful estimates of turbulent velocities,  $\sigma_w$  and  $\sigma_v$ , when the inputs are mean wind speeds, temperatures and temperature fluctuations measured with a tower located in an urban area? This paper examines the performance of these methods during unstable as well as stable conditions. Furthermore, in view of uncertainties associated with methods that rely on differences between two levels, we confine our examination to methods that use measurements at a single level.

# 2. Governing equations

Under stable conditions, the method examined in this paper is based on fitting measurements of mean wind and temperature to profiles given by MO similarity theory (Businger, 1973):

$$u(z_{\rm r}) = \frac{u_*}{k} \left( \ln \left( \frac{z_{\rm r}}{z_0} \right) + \beta \frac{(z_{\rm r} - z_0)}{L} \right),\tag{1}$$

where  $u_*$  is the friction velocity, k is the von Karman constant,  $z_r$  is the height at which the mean wind, u, is measured,  $z_0$  is the surface roughness length,  $\beta = 4.7$  is a constant, and L is the MO length defined by

$$L = -\frac{T_0}{g} \frac{u_*^3}{k w'\theta'} = \frac{T_0}{g} \frac{u_*^2}{k\theta_*}.$$
 (2)

Here, g presents the acceleration due to gravity,  $\overline{w'\theta'}$  is the kinematic heat flux calculated as correlation of the fluctuations of vertical velocity, w', and potential temperature  $\theta'$ ,  $\theta_* = -\overline{w'\theta'}/u_*$  is the temperature scale, and  $T_0$  is a reference temperature. The height,  $z_r$ , in the equation is measured relative to a displacement height, d.

The SU (stands for single U) method examined here uses the mean wind measured at a single height and estimates of the roughness and displacement heights. The first version of the method, is based on the empirical observation (Venkatram, 1980), based on measurements made in Kansas (Izumi, 1971), Minnesota (Caughey et al., 1979), and Prairie Grass (Barad, 1958), that  $\theta_*$  varies little with  $u_*$ , so that  $L \sim u_*^2$ . Useful estimates of L and  $u_*$  can be obtained by taking  $\theta_*$  to be 0.08 °C. Van Ulden and Holtslag (1985) provide theoretical support for the small variation of  $\theta_*$  around the value used in this study. We realize that there is no a priori justification for using a value of  $\theta_*$  determined in experiments conducted in rural areas to urban surfaces.

If we take  $\theta_*$  to be a constant in Eq. (2), and substitute for L in Eq. (1), we obtain a quadratic equation for  $u_*$ , which yields:

$$u_* = C_{\rm D} u(z_{\rm r}) \left\{ \frac{1}{2} + \frac{1}{2} \left[ 1 - \left( \frac{2u_0}{C_{\rm D}^{1/2} u} \right)^2 \right]^{1/2} \right\}, \qquad (3)$$

where  $C_{\rm D} = k/\ln(z_{\rm r}/z_0)$ ,  $u_0^2 = \beta(z_{\rm r} - z_0)/kA_{\rm L}$ , and  $A_{\rm L} = T_0/(gk\theta_*)$ . When the term  $2u_0/C_{\rm D}^{1/2}u$  within the square root sign exceeds unity, the surface friction velocity is computed from

$$u_* = \frac{1}{2}C_{\rm D}u(z_{\rm r}). \tag{4}$$

The values of  $\sigma_w$  and  $\sigma_v$  are computed from the similarity relationships:

$$\sigma_{\rm w} = 1.6u_* \quad \text{and} \quad \sigma_{\rm v} = 1.9u_*. \tag{5}$$

The second version of the SU method estimates  $\theta_*$ from measurements of the standard deviation of temperature fluctuations,  $\sigma_T$ , which in principle can be measured with inexpensive thermistors. The relationship between the two variables is taken to be (Stull, 1988):

$$\theta_* = 0.5\sigma_{\rm T}.\tag{6}$$

In principle, Eq. (6) should yield better results than assuming a constant  $\theta_*$ .

Under unstable conditions, we use the similarity relationship for free convection

$$\frac{\sigma_{\rm T}}{\theta_*} = -0.95 \left(-\frac{z_{\rm r}}{L}\right)^{1/3} \tag{7}$$

to express the kinematic surface heat flux,  $Q_0 = w'\theta'$ , in terms of  $\sigma_T$ 

$$Q_0 = \left(\frac{\sigma_{\rm T}}{0.95}\right)^{3/2} \left(\frac{g\kappa z_{\rm r}}{T_0}\right)^{1/2}.$$
(8)

The heat flux can then be used to calculate the surface friction velocity using an approximation

proposed by Wang and Chen (1980):

$$u_* = \kappa u \frac{1 + d_1 \ln(1 + d_2 d_3)}{\ln(1/r_{\rm h})},\tag{9}$$

where

$$r_{\rm h} = \frac{z_0}{z_{\rm r} - d}$$

$$d_1 = \begin{cases} 0.128 + 0.005 \ln(r_{\rm h}), & \text{for } r_{\rm h} \leq 0\\ 0.107, & \text{otherwise} \end{cases}$$
(10)
$$d_2 = 1.95 + 32.6r_{\rm h}^{0.45}$$

$$d_3 = \frac{Q_0 \kappa g(z_{\rm r} - d)}{T_0}.$$

The standard deviation of the vertical velocity fluctuations,  $\sigma_w$ , is computed from

$$\sigma_{\rm w} = (\sigma_{\rm ws}^3 + \sigma_{\rm wc}^3)^{1/3},\tag{11}$$

where the shear component,  $\sigma_{ws}$ , is taken to be

$$\sigma_{\rm ws} = 1.3u_* \tag{12}$$

and the convective component,  $\sigma_{wc}$ , is

$$\sigma_{\rm wc} = \begin{cases} 1.3 \left(\frac{g}{T_0} Q_0 z_{\rm r}\right)^{1/3} = 1.3 u_{\rm f} & \text{for } z_{\rm r} \le 0.1 z_i \\ 0.6 w_* & \text{for } z_{\rm r} \ge 0.1 z_i \end{cases}$$
(13)

The free convection velocity scale,  $u_{\rm f}$ , is dependent on the reference height,  $z_{\rm r}$ . The convective velocity scale  $w_*$  is

$$w_* = \left(\frac{g}{T_0} Q_0 z_i\right)^{1/3}.$$
 (13a)

The standard deviation of the horizontal velocity fluctuations  $\sigma_v$  is computed from

$$\sigma_{\rm v} = (\sigma_{\rm vs}^3 + \sigma_{\rm vc}^3)^{1/3}, \tag{14}$$

where the shear component,  $\sigma_{vs}$ , is calculated as

$$\sigma_{\rm vs} = 1.9u_*,\tag{15}$$

and the convective component,  $\sigma_{vc}$ , is taken to be

$$\sigma_{\rm vc} = 0.6w_*.\tag{16}$$

The convective velocity,  $w_*$ , depends on the mixed layer height,  $z_i$ , the estimation of which is discussed for each site independently in the following sections.

### 3. Field observations

Measurements used in this study were drawn from Wilmington 2005 (Yuan et al., 2006), and VTMX 2000 (Monti et al., 2002) field studies. The application of the preceding methods to each of these sites is discussed next.

# 3.1. Wilmington study

The Wilmington field study, sponsored by the California Air Resources Board and California Energy Commission, took place in the city of Wilmington, California, during the summer months of 2004 and 2005. Wilmington is a small community located next to the Port of Los Angeles. It is surrounded by numerous small industries, transportation corridors, and port businesses, which are located to the south of residential areas. The instruments used in this analysis were deployed at the Harbor Generating Station of the City of Los Angeles's Department of Water and Power (LADWP). The residential areas, consisting mostly of one storey buildings about 4 m high, are located upwind of the LADWP site during the dominant nighttime, stable, northwesterly flows.

The meteorological instrumentation deployed at LADWP consisted of mini-Sodar, net-radiometer, krypton hygrometer, IR surface temperature measurements, thermistors, RH probe and two sonic anemometers mounted at 3.1 and 6 m above ground level (agl). The sonic anemometers were used to measure three velocity components (resolution and accuracy: 0.01 and  $\pm 0.05 \text{ m s}^{-1}$ ) and virtual air temperature (resolution and accuracy: 0.01 and  $\pm 0.05 \text{ s}^{-1}$ ) and  $\pm 0.05 \text{ °C}$ ) at 10 Hz.

The analysis that follows is based on 5 min averaged data from the sonic anemometer at a height of 6 m. As described in Princevac and Venkatram (2007), we first determined the roughness length,  $z_0$ , empirically by fitting the similarity wind profile, Eq. (1), to the observed wind data using values of  $u_*$  and L calculated from surface shear stress and heat flux measurements. To account for the effects of the buildings upwind of the site, we incorporated a displacement height, d, in the similarity profiles using a constant ratio of  $d/z_0 = 5$ based on a recommendation in Britter and Hanna (2003).

Using estimates of roughness and displacement heights (see Princevac and Venkatram, 2007) the friction velocity was determined using Eqs. (3)



Fig. 1. Comparison of estimated  $\sigma_w$  (Eq. (5)) with observed values: (a) assuming that  $\theta_*$  is a constant and (b) using  $\theta_*$  derived from measured  $\sigma_T$  according to Eq. (6).



Fig. 2. Comparison of estimated  $\sigma_v$  with observed values: (a) assuming that  $\theta_*$  is a constant and (b) using  $\theta_*$  derived from measured  $\sigma_T$ .

and (4) with constant  $\theta_* = 0.08$  °C. The standard deviation of the vertical velocity fluctuations was estimated with Eq. (5) and is presented in Fig. 1a.

Fig. 1a indicates that a single measurement of wind speed provides an adequate estimate of  $\sigma_w$  when the estimated values are  $> 0.05 \,\mathrm{m \, s^{-1}}$ . The observed values do not drop below  $0.06 \,\mathrm{m \, s^{-1}}$  when the estimated values become much smaller. Fig. 1b

compares the  $\sigma_w$  estimated with Eqs. (3)–(5) with observed values, but this time  $\theta_*$  is calculated from Eq. (6). We see that using  $\sigma_T$  to compute  $\theta_*$  does not improve the comparison in this case.

Fig. 2a and b compares estimates of  $\sigma_v$  with and without using  $\sigma_T$  with corresponding observations. In this particular case, assuming a constant  $\theta_*$  results in slightly better comparison with observed values.

Under convective conditions, it is necessary to estimate the height of the thermal internal boundary layer (TIBL) that forms when southerly flow during the daytime brings air from the ocean onto the warmer land in Wilmington. The TIBL height,  $z_i$ , is computed using the expression (Venkatram, 1977):

$$z_{i} = a \left(\frac{Q_{0}(x+x_{0})}{U\gamma}\right)^{1/2},$$
(17)

where  $Q_0$  is the average kinematic heat flux over land, x is the distance from the shoreline, U is the boundary layer averaged wind speed, a is empirically determined parameter set to be 2, and  $\gamma$  is the potential temperature gradient above the TIBL. The parameter  $x_0$  is the distance of the effective shoreline from the release. Taking  $x_0 = 100$  m yielded the best agreement between modeled and observed mixed layer heights at locations where it was measured.

Then, an expression for  $w_*$  can be obtained by combining Eqs. (13a) and (17)

$$w_* = Q_0^{1/2} \left(\frac{g}{T_0}\right)^{1/3} \left(\frac{\alpha(x+x_0)}{U\gamma}\right)^{1/6}.$$
 (18)

This estimate is used in Eqs. (13)–(16) to compute the standard deviations of the vertical,  $\sigma_w$ , and horizontal velocity fluctuations,  $\sigma_v$ .

Fig. 3a compares estimates of the surface heat flux from Eq. (8) with corresponding observed values. The comparison is excellent with Eq. (8) explaining 70% of the observed variance of the heat

flux, and over 90% of the observed values within a factor of two of the estimates.

Fig. 3b indicates that Eq. (9) in combination with an estimate of the heat flux from Eq. (8) provides adequate estimates of the surface friction velocity  $u_*$ . Fig. 4a shows that estimates of  $\sigma_w$  by Eqs. (11)–(13) compare well with observations, although there is a tendency to overestimate. Eqs. (14)–(16) yield estimates of  $\sigma_v$  that explain 65% of the observed variance, and 97% of the observations are within a factor of two of the model estimates as presented in Fig. 4b.

The results presented in this section indicate that wind speed and  $\sigma_T$  measured at a single height can provide estimates of variables required to model dispersion in the surface layer in an urban area. We next examine the applicability of these measurements to a second urban site.

# 4. VTMX study

The VTMX, sponsored by the US Department of Energy, took place in the SLC metropolitan area in October 2000. The SLC metropolitan area is located in a wide valley  $\sim$ 1400 m above the mean sea level (msl). The valley is about 30 km wide (along the East–West direction) and 50 km long (North–South direction) and is surrounded by elevated mountains (up to 3000 m above the msl). The southern shoreline of the Great Salt Lake is the northwestern border of the valley. Data used in this analysis was



Fig. 3. Comparison of observed: (a)  $Q_0$  values with estimates from measured  $\sigma_T$  in Eq. (8) and (b)  $u_*$  values with estimates from Eq. (9).



Fig. 4. Comparison of observed (a)  $\sigma_w$  with estimates from Eqs. (11)–(13) and (b) observed  $\sigma_v$  with estimates from Eqs. (13)–(16). Here,  $z_i$  was estimated using  $x_0 = 100$  and x = 1000 in Eq. (17).

collected by the Arizona State University's Environmental Fluid Dynamics Program at the Arizona Cemetery Site (ACS). The ACS was located in the northeastern side of the valley, in a grassy open area (aerodynamic roughness length <0.1 m), with a gentle slope ( $\sim 0.07$ , i.e.,  $4^{\circ}$ ). Because the measurements were made away from buildings and trees, the data can be considered free from the immediate effects of obstacle wakes. The fetch was fairly uniform for 100 m uphill and 80 m downhill distances. The closest uphill feature was the Utah National Guards' Building (10 m high), but it was not in the direct downslope path of the nighttime stable katabatic flow through the measurement station. The closest downstream feature was a mild drop in slope to accommodate a football stadium and a school with a building height of approximately 15m which is not expected to have any upstream influence. On a much larger (~km) scale, the major topographic perturbation was provided by the Wasatch Mountain range abutting the gentle slope. More details on VTMX campaign can be found in Monti et al. (2002) and Doran et al. (2002).

The meteorological instruments deployed at the ACS consisted of a 14 m mast equipped with cup anemometers, thermistors, an upward facing spectral pyranometer, a downward facing pyrgeometer, and two sonic anemometers-thermometers placed at 4.5 and 13.86 m above ground level. Also, two tethered systems were deployed at the site to analyze the vertical structure of the lower atmosphere. For

this study, only the data from the sonic anemometers were utilized. The wind speeds and the  $\sigma_{T}$  at the 4.5 m level were used to estimate dispersion variables.

Fig. 5a indicates that a single measurement of wind speed provides an adequate estimate of  $\sigma_w$  when the estimated values are  $> 0.05 \,\mathrm{m \, s^{-1}}$ . As in the case of Wilmington, the observed values do not drop below  $0.06 \,\mathrm{m \, s^{-1}}$  when the estimated values become much smaller. Fig. 5b shows that using  $\sigma_T$  to compute  $\theta_*$  improves the results slightly.

Fig. 6a and b indicates that although estimates of  $\sigma_v$  are not well correlated with observed values, close to 80% of the observations are within a factor of two of the model estimates. The observed values of  $\sigma_v$  are rarely below  $0.2 \text{ m s}^{-1}$  when the estimated values are much lower. This suggests the presence of mesoscale meandering that is not captured by MO similarity estimates based on the surface friction velocity.

As in the case of Wilmington, Fig. 7a shows that Eq. (8), based on free convection similarity, provides adequate estimates of heat flux: it explains 50% of the observed variance, and 84% of the observations are within a factor of two of the estimated values. Fig. 7b shows that Eqs. (8) and (9) explain 33% of the observed variance of the friction velocity,  $u_*$ , and 85% of the observations are within a factor of two of the estimated values.

Fig. 8a and b shows estimates of  $\sigma_w$  and  $\sigma_v$  from Eqs. (11)–(16), where the mixed layer height



Fig. 5. Comparison of estimated  $\sigma_w$  with observed values: (a) assuming that  $\theta_*$  is a constant and (b) using  $\theta_*$  derived from measured  $\sigma_T$ .



Fig. 6. Comparison of estimated  $\sigma_v$  with observed values: (a) assuming that  $\theta_*$  is a constant and (b) using  $\theta_*$  derived from measured  $\sigma_T$ .

was computed from the one-dimensional boundary layer equation:

$$z_i^2(T) = \frac{2}{\gamma} \int_0^T Q_0(t) \,\mathrm{d}t,\tag{19}$$

where  $\gamma$ , is the potential temperature gradient above the mixed layer, was assigned a nominal value of 10 K/1000 m.

Fig. 8a shows that Eqs. (11)–(13) to estimate  $\sigma_w$  explain 32% of the observed variance, and over 90% of the observations are within a factor of two

of the estimates. The estimates of  $\sigma_v$  (Fig. 8b) explain 30% of the observed variance, and over 80% of the observations are within a factor of two of the estimated values.

### 5. Conclusions and discussion

The results from this study indicate as follows:

1. Measurements of wind speed and standard deviation of temperature fluctuations,  $\sigma_{T}$ , at



Fig. 7. Comparison of observed (a)  $Q_0$  values with estimates from measured  $\sigma_T$  in Eq. (8) and (b) observed  $u_*$  values with estimates from Eq. (9).



Fig. 8. Comparison of observed (a)  $\sigma_w$  with estimates from Eqs. (11)–(13) and (b)  $\sigma_v$  with estimates from Eqs. (13)–(16).

one level can yield useful estimates of parameters required to model dispersion in urban areas. Under stable conditions, using  $\sigma_T$  to estimate  $\theta_*$ does not improve the results over those based on a nominal value of  $\theta_* = 0.08$  °C.

- 2. Under unstable conditions, Eq. (8), based on free convection theory, provides excellent estimates of surface heat flux using measured  $\sigma_{\rm T}$  as an input.
- 3. These estimates of heat flux when used in MO similarity equations provide estimates of surface friction velocity and standard deviations of

turbulent velocities that are within a factor of two of the observations over 80% of the time.

The results from this study indicate that the MO theory can be used to interpret measurements of mean wind speed and standard deviation of temperature fluctuations at a single level on a tower located in an urban area to yield variables required for dispersion modeling. The methods described in this paper require independent estimates of roughness and displacement heights, which in principle can be obtained with models (Grimmond and Oke, 1999) that use building morphology as inputs. The micrometeorological variables inferred from tower measurements correspond to the immediate location of the tower. Because these variables are expected to vary spatially in an urban area, it might be necessary to make several measurements to obtain input values for dispersion models. These values can be used directly in a dispersion model that explicitly accounts for their spatial variation or averaged to obtain representative values for inputs to a dispersion model that assumes horizontal homogeneity.

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#### References

- Barad, M.L. (Ed.), 1958. Project Prairie Grass. A Field Program in Diffusion. Geophysical Research Paper No. 59, vols. I and II, AFCRF-TR-58-235, Air Force Cambridge Research Center, Bedford, MA.
- Britter, R.E., Hanna, S.R., 2003. Flow and dispersion in urban areas. Annual Review of Fluid Mechanics 35, 469–496.
- Businger, J.A., 1973. Turbulent transfer in the atmospheric surface layer. In: Haugen, D.A. (Ed.), Workshop on Micrometeorology. American Meteorological Society, pp. 67–100.

- Caughey, S.J., Wyngaard, J.C., Kaimal, J.C., 1979. Turbulence in the evolving stable boundary layer. Journal of Atmospheric Science 36, 1041–1052.
- Cimorelli, A.J., Perry, S.G., Venkatram, A., Weil, J.C., Paine, R.J., Wilson, R.B., Lee, R.F., Peters, W.D., Brode, R.W., 2005. AERMOD: a dispersion model for industrial source applications. Part I: General model formulation and boundary layer characterization. Journal of Applied Meteorology 44, 682–693.
- Doran, J.C., Fast, J.D., Horel, J., 2002. The VTMX 2000 campaign. Bulletin of the American Meteorological Society 83 (4), 537–554.
- Grimmond, C.S.B., Oke, T.R., 1999. Aerodynamic properties of urban areas derived, from analysis of surface form. Journal of Applied Meteorology 38 (9), 1262–1292.
- Izumi, Y., 1971. Kansas 1968 Field Program Data Report, Environmental Research Papers No. 379, AFCRL-72-0041, Air Force Cambridge Research Laboratories, Bedford, MA.
- Monti, P., Fernando, H.J.S., Princevac, M., Chan, W.C., Kowalewski, T.A., Pardyjak, E.R., 2002. Observations of flow and turbulence in the nocturnal boundary layer over a slope. Journal of Atmospheric Science 59, 2513–2534.
- Princevac, M., Venkatram, A., 2007. Estimating micrometeorological inputs for modeling dispersion in urban areas during stable conditions. Atmospheric Environment 41 (26), 5345–5356.
- Stull, R.B., 1988. An Introduction to Boundary Layer Meteorology. Kluwer Academic Publishers, Dordrecht, 666pp.
- Van Ulden, A.P., Holtslag, A.A.M., 1985. Estimation of atmospheric boundary-layer parameters for diffusion applications. Journal of Climate and Applied Meteorology 24, 1196–1207.
- Venkatram, A., 1977. Internal boundary-layer development and fumigation. Atmospheric Environment 11 (5), 479–482.
- Venkatram, A., 1980. Estimating the Monin–Obukhov length in the stable boundary layer for dispersion calculations. Boundary-Layer Meteorology 19, 481–485.
- Wang, I.T., Chen, P.C., 1980. Estimation of heat and momentum fluxes near the ground. In: Proceedings of the 2nd Joint Conference on Applications on Air Pollution Meteorology, AMS, 45 Beacon Street, Boston, MA, pp. 764–769.
- Yuan, J., Venkatram, A., Isakov, V., 2006. Dispersion from ground-level sources in a shoreline urban area. Atmospheric Environment 40, 1361–1372.